Solutions to selected exercises

Rabe-Hesketh, S. and Skrondal, A. (2021). ***Multilevel and Longitudinal Modeling Using Stata (4th Edition).*** **College Station, TX: Stata Press.**

Volume II: Categorical Responses, Counts, and Survival

Contents

Disclaimer

We have solved the exercises as well as we could but there may be better solutions and we may have made mistakes. We are grateful for any suggestions for improvement.

Please also check the errata at http://www.stata.com/bookstore/mlmus4.html for any errors in the wording of the exercises themselves.

10.3 Vaginal-bleeding data

1. *Produce an identifier variable for women, and reshape the data to long form, stacking the responses* `y1`–`y4` *into one variable and creating a new variable,* `occasion`*, taking the values 1–4 for each woman.*

```
. use amenorrhea, clear
. generate id = _n
. reshape long y, i(id) j(occasion)
(note: j = 1 2 3 4)
Data                               wide   ->   long
-----------------------------------------------------------------------------
Number of obs.                       57   ->     228
Number of variables                   7   ->       5
j variable (4 values)                     ->   occasion
xij variables:
                           y1 y2 ... y4   ->   y
-----------------------------------------------------------------------------
```

2. *Fit the following model considered by Fitzmaurice, Laird, and Ware (2011):*

$$\text{logit}\{\Pr(y_{ij}=1|x_j,t_{ij},\zeta_j)\} \;=\; \beta_1+\beta_2 t_{ij}+\beta_3 t_{ij}^2+\beta_4 x_j t_{ij}+\beta_5 x_j t_{ij}^2+\zeta_j$$

where $t_{ij}=1,2,3,4$ *is the time interval and* x_j *is* `dose`*. It is assumed that* $\zeta_j \sim N(0,\psi)$*, and that* ζ_j *is independent across women and independent of* x_j *and* t_{ij}*. Use* `melogit` *with the* `fweight(wt2)` *option to specify that* `wt2` *are level-2 frequency weights.*

```
. generate time = occasion
. generate dose_time = dose*time
. generate time2 = time^2
. generate dose_time2 = dose*time2
. melogit y time time2 dose_time dose_time2 || id:, intpoints(30) fweight(wt2)
Mixed-effects logistic regression               Number of obs     =      3,616
Group variable:              id                 Number of groups  =      1,151

                                                Obs per group:
                                                              min =          1
                                                              avg =        3.1
                                                              max =          4

Integration method: mvaghermite                 Integration pts.  =         30

                                                Wald chi2(4)      =     291.00
Log likelihood =  -1934.465                     Prob > chi2       =     0.0000
------------------------------------------------------------------------------
           y |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
        time |   1.133202   .2682216     4.22   0.000     .6074974    1.658907
       time2 |  -.0419232   .0548099    -0.76   0.444    -.1493486    .0655022
   dose_time |   .5644407   .1922395     2.94   0.003     .1876583    .9412231
  dose_time2 |  -.1095528   .0496097    -2.21   0.027     -.206786   -.0123195
       _cons |  -3.805677   .3049807   -12.48   0.000    -4.403428   -3.207926
-------------+----------------------------------------------------------------
id           |
   var(_cons)|   5.064584   .5840171                      4.040065     6.34891
------------------------------------------------------------------------------
LR test vs. logistic model: chibar2(01) = 500.52      Prob >= chibar2 = 0.0000
```

3. *Write down the above model, adding a random slope of t_{ij}, and fit the model. (See section 11.7.1 for an example of a random-coefficient model for ordinal responses fit in* `meologit`*.)*

$$\text{logit}\{\Pr(y_{ij}=1|x_j,t_{ij},\zeta_j)\} \;=\; \beta_1+\beta_2 t_{ij}+\beta_3 t_{ij}^2+\beta_4 x_j t_{ij}+\beta_5 x_j t_{ij}^2+\zeta_{1j}+\zeta_{2j}t_{ij},$$

where ζ_{1j} and ζ_{2j} are a random intercept and random slope of time, and are assumed to have a bivariate normal distribution with zero means, variances ψ_1 and ψ_2 and correlation ρ.

```
. melogit y time time2 dose_time dose_time2 || id: time,
>  covariance(unstructured) intpoints(30) fweight(wt2)
Mixed-effects logistic regression               Number of obs     =      3,616
Group variable:              id                 Number of groups  =      1,151

                                                Obs per group:
                                                              min =          1
                                                              avg =        3.1
                                                              max =          4

Integration method: mvaghermite                 Integration pts.  =         30

                                                Wald chi2(4)      =     147.49
Log likelihood = -1927.0875                     Prob > chi2       =     0.0000

------------------------------------------------------------------------------
           y |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
        time |   .8161103     .34782     2.35   0.019     .1343956    1.497825
       time2 |   .0175889   .0662461     0.27   0.791     -.112251    .1474288
   dose_time |   .5479068    .197515     2.77   0.006     .1607846    .9350291
  dose_time2 |  -.0989771   .0534594    -1.85   0.064    -.2037555    .0058013
       _cons |  -3.448329   .4566772    -7.55   0.000      -4.3434   -2.553258
-------------+----------------------------------------------------------------
id           |
   var(time)|   .5104948   .2012609                      .2357262    1.105541
  var(_cons)|   4.656183   1.710134                      2.266724    9.564482
-------------+----------------------------------------------------------------
id           |
 cov(time,_cons)|  -.335804   .4233619    -0.79   0.428    -1.165578      .49397
------------------------------------------------------------------------------
LR test vs. logistic model: chi2(3) = 515.27              Prob > chi2 = 0.0000

Note: LR test is conservative and provided only for reference.
```

4. *Interpret the estimated coefficients.*

The model assumes that there is no difference in the log-odds of amenorrhea between the groups at time 0 (baseline). In the low-dose group, the log-odds increase approximately by the same amount of 0.82 in each 3-month interval (since the estimated coefficient of `time2` is small and nonsignificant), corresponding to an odds ratio of about 2.3. The interaction between `dose` and `time2` is not quite significant, so we could assume a linear relationship for both group by removing the terms `dose_time2` and `time2`. However, keeping the terms in, the high-dose group initially has a larger slope than the low-dose group, and the slope decreases over time because time-squared has a negative coefficient ($.0176 - .0990$).

5. *Plot marginal predicted probabilities as a function of time, separately for women in the two treatment groups.*

```
. predict prob, pr marginal
(using 30 quadrature points)
. sort dose id time
. twoway (line prob time if dose==0, sort) (line prob time if dose==1, sort),
>  ytitle(Predicted marginal probability) xtitle(Time in 90 day intervals)
>  legend(order(1 "Low dose" 2 "High dose"))
```

The graph is shown in figure 1.

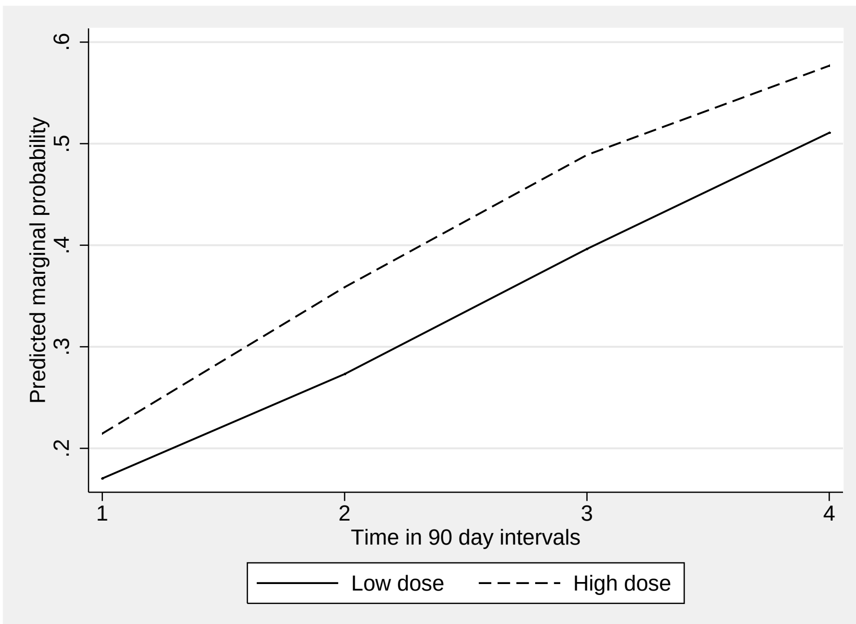

Figure 1: Predicted marginal probabilities over time by dose level

10.8 PISA data

1. *Fit a logistic regression model with* `pass_read` *as the response variable and the variables* `female` *to* `both_for` *above as covariates and with a random intercept for schools using* `melogit`. *(Use the default seven quadrature points.)*

```
. use pisaUSA2000, clear

. melogit pass_read female isei high_school college test_lang
>    one_for both_for || id_school:

Mixed-effects logistic regression               Number of obs     =      2,069
Group variable:       id_school                 Number of groups  =        148

                                                Obs per group:
                                                              min =          1
                                                              avg =       14.0
                                                              max =         28

Integration method: mvaghermite                 Integration pts.  =          7

                                                Wald chi2(7)      =     116.85
Log likelihood = -1252.8108                     Prob > chi2       =     0.0000

------------------------------------------------------------------------------
   pass_read |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
      female |   .5422162    .103192     5.25   0.000     .3399635    .7444688
        isei |   .0206763    .003284     6.30   0.000     .0142398    .0271129
 high_school |   .4447944   .2565114     1.73   0.083    -.0579586    .9475475
     college |    .796881    .255052     3.12   0.002     .2969883    1.296774
   test_lang |   .7825093   .2834799     2.76   0.006     .2268988     1.33812
     one_for |   .0112567   .2244283     0.05   0.960    -.4286147    .4511282
    both_for |    .150784   .2376408     0.63   0.526    -.3149834    .6165514
       _cons |  -3.279323   .3811204    -8.60   0.000    -4.026305   -2.532341
-------------+----------------------------------------------------------------
id_school    |
   var(_cons)|   .5134392   .1283984                      .3145029    .8382111
------------------------------------------------------------------------------
LR test vs. logistic model: chibar2(01) = 58.35       Prob >= chibar2 = 0.0000
```

2. *Fit the model from step 1 with the school mean of* `isei` *as an additional covariate.*

```
. egen mn_isei = mean(isei), by(id_school)
```

(*Continued on next page*)

```
. melogit pass_read female isei mn_isei high_school college test_lang
>    one_for both_for || id_school:

Mixed-effects logistic regression               Number of obs     =      2,069
Group variable:       id_school                 Number of groups  =        148

                                                Obs per group:
                                                              min =          1
                                                              avg =       14.0
                                                              max =         28

Integration method: mvaghermite                 Integration pts.  =          7

                                                Wald chi2(8)      =     171.58
Log likelihood = -1225.4697                     Prob > chi2       =     0.0000

------------------------------------------------------------------------------
   pass_read |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
      female |   .5552102    .102912     5.39   0.000     .3535063    .7569141
        isei |   .0143423    .003335     4.30   0.000     .0078058    .0208787
     mn_isei |   .0690722   .0092476     7.47   0.000     .0509472    .0871971
 high_school |   .3999543   .2561423     1.56   0.118    -.1020754    .9019839
     college |   .7207869    .254843     2.83   0.005     .2213038     1.22027
   test_lang |   .6951881   .2849896     2.44   0.015     .1366188    1.253757
     one_for |  -.0199176   .2239413    -0.09   0.929    -.4588344    .4189992
    both_for |   .0986699   .2359626     0.42   0.676    -.3638082     .561148
       _cons |   -6.03362   .5387266   -11.20   0.000    -7.089505   -4.977736
-------------+----------------------------------------------------------------
id_school    |
   var(_cons)|   .2714333   .0857003                      .1461878    .5039822
------------------------------------------------------------------------------
LR test vs. logistic model: chibar2(01) = 25.15       Prob >= chibar2 = 0.0000
```

3. *Interpret the estimated coefficients of* `isei` *and school mean* `isei` *and comment on the change in the other parameter estimates due to adding school mean* `isei`.

Within a school, student's ISEI score has an estimated effect of 0.014 on the log-odds scale and between schools there is an additional effect of 0.069. Considering a 10-unit change in ISEI, the corresponding odds ratios are 1.15 ($= \exp(0.14)$) and 2.00 ($= \exp(0.69)$). Comparing two students from the same school, one of whom has ISEI 10 points higher than the other (with all other covariates being the same), the higher ISEI student has a 15% greater odds of passing the reading test. Comparing two students with the same ISEI score (and other covariate values) from schools that differ in their mean ISEI score by 10 units (but have the same random intercept), the student from the higher mean ISEI school has twice the odds of passing the reading test as the other student.

The estimated random intercept variance has nearly halved due to adding school mean ISEI. The estimates of the effects of parent's education on test language spoken at home have decreased a little.

4. *From the estimates in step 2, obtain an estimate of the between-school effect of socioeconomic status.*

The total between-school effect on the log-odds scale is the sum of the coefficient of `isei` and `mn_isei`, giving 0.083 ($= 0.014 + 0.069$).

5. *Rerun the command but this time with robust standard errors.*

```
. melogit pass_read female isei mn_isei high_school college test_lang
>    one_for both_for || id_school:, vce(robust)

Mixed-effects logistic regression               Number of obs     =      2,069
Group variable:       id_school                 Number of groups  =        148

                                                Obs per group:
                                                              min =          1
                                                              avg =       14.0
                                                              max =         28

Integration method: mvaghermite                 Integration pts.  =          7

                                                Wald chi2(8)      =     188.38
Log pseudolikelihood = -1225.4697               Prob > chi2       =     0.0000
                            (Std. Err. adjusted for 148 clusters in id_school)
```

pass_read	Coef.	Robust Std. Err.	z	P>|z|	[95% Conf.	Interval]
female	.5552102	.1024602	5.42	0.000	.354392	.7560285
isei	.0143423	.0029873	4.80	0.000	.0084873	.0201973
mn_isei	.0690722	.0090417	7.64	0.000	.0513507	.0867936
high_school	.3999543	.2619124	1.53	0.127	-.1133846	.9132932
college	.7207869	.2574594	2.80	0.005	.2161757	1.225398
test_lang	.6951881	.2694431	2.58	0.010	.1670892	1.223287
one_for	-.0199176	.1998362	-0.10	0.921	-.4115894	.3717542
both_for	.0986699	.2452363	0.40	0.687	-.3819845	.5793243
_cons	-6.03362	.5471279	-11.03	0.000	-7.105971	-4.961269
id_school						
var(_cons)	.2714333	.0815196			.1506682	.4889951

The robust and model-based standard errors are quite similar in this case.

6. ❖ *In this survey, schools were sampled with unequal probabilities, π_j, and given that a school was sampled, students were sampled from the school with unequal probabilities $\pi_{i|j}$. The reciprocals of these probabilities are given as school- and student-level survey weights,* `wnrschbg` *($w_j = 1/\pi_j$) and* `w_fstuwt` *($w_{i|j} = 1/\pi_{i|j}$), respectively. As discussed in Rabe-Hesketh and Skrondal (2006), incorporating survey weights in multilevel models using a so-called pseudolikelihood approach can lead to biased estimates, particularly if the level-1 weights $w_{i|j}$ are very different from 1 and if the cluster sizes are small. Neither of these issues arise here, so implement pseudo* ML *estimation as follows:*

   a. *Rescale the student-level weights by dividing them by their cluster means [this is scaling method 2 in Rabe-Hesketh and Skrondal (2006)].*

```
. egen mnw = mean(w_fstuwt), by(id_school)
. generate wt1 = w_fstuwt/mnw
```

   b. *Rename the level-2 weights and rescaled level-1 weights to* `wt2` *and* `wt1`*, respectively.*

```
. rename wnrschbw wt2
```

   c. *Run the* `melogit` *command from step 2 above, adding* `[pw=wt1]` *before* `||` *to specify level-1 weights and giving the additional option* `pweight(wt2)` *to specify level-2 weights.*

```
. melogit pass_read female isei mn_isei high_school college test_lang
>    one_for both_for [pw=wt1] || id_school:, pweight(wt2)
Mixed-effects logistic regression               Number of obs     =      2,069
Group variable:       id_school                 Number of groups  =        148
                                                Obs per group:
                                                              min =          1
                                                              avg =       14.0
                                                              max =         28
Integration method: mvaghermite                 Integration pts.  =          7
                                                Wald chi2(8)      =      88.21
Log pseudolikelihood = -197964.36               Prob > chi2       =     0.0000
                             (Std. Err. adjusted for 148 clusters in id_school)
```

pass_read	Coef.	Robust Std. Err.	z	P>|z|	[95% Conf.	Interval]
female	.6218815	.1540693	4.04	0.000	.3199113	.9238518
isei	.0182009	.0048055	3.79	0.000	.0087823	.0276194
mn_isei	.0682412	.0164298	4.15	0.000	.0360394	.1004431
high_school	.1019583	.4766682	0.21	0.831	-.8322941	1.036211
college	.4528053	.5050718	0.90	0.370	-.5371173	1.442728
test_lang	.6245946	.3825914	1.63	0.103	-.1252707	1.37446
one_for	-.1086342	.274045	-0.40	0.692	-.6457526	.4284843
both_for	-.2811825	.3265266	-0.86	0.389	-.9211629	.3587979
_cons	-5.875258	.9545544	-6.15	0.000	-7.74615	-4.004366
id_school						
var(_cons)	.2962084	.124311			.1301279	.6742557

d. *Compare the estimates with those from step 2. Robust standard errors are computed by* `melogit` *because model-based standard errors are not appropriate with survey weights.*

   Some of the estimates are quite different, especially the coefficients of `high_school` and `college`.

11.7 Recovery-after-surgery data

1. *Reshape the data to long form, stacking the recovery scores at the four occasions into a single variable and generating an identifier,* `occ`*, for the four occasions. (You can specify several variables in the* `i()` *option of the* `reshape` *command if one variable does not uniquely identify the individuals.) Recode the recovery score to four categories (to simplify some of the commands below), by merging {0,1}, {2,3}, and {4,5} and calling the new categories 1, 2, 3, and 4.*

```
. use recovery, clear
. reshape long score, i(id dosage) j(occ)
(note: j = 1 2 3 4)
Data                               wide   ->   long
-----------------------------------------------------------------------------
Number of obs.                       60   ->     240
Number of variables                   8   ->       6
j variable (4 values)                     ->   occ
xij variables:
               score1 score2 ... score4   ->   score
-----------------------------------------------------------------------------
```

   Before we forget, let us construct a unique person identifier

```
. egen id2 = group(id dosage)
```

   Now recode the response variable:

```
. recode score 0/1=1 2/3=2 4/5=3 6=4
(score: 164 changes made)
```

2. *Construct a variable,* `time`*, taking the values 0, 5, 15, and 30 at the four occasions. Fit a random-intercept proportional-odds model* `meologit` *with dummy variables for the dosage groups and the continuous variables* `age`*,* `duration`*, and* `time` *as covariates. (Make sure there are 60 level-2 clusters.)*

```
. recode occ 1=0 2=5 3=15 4=30, generate(time)
(240 differences between occ and time)
. tabulate dosage, generate(dose)
     dosage |      Freq.     Percent        Cum.
------------+-----------------------------------
         15 |         60       25.00       25.00
         20 |         60       25.00       50.00
         25 |         60       25.00       75.00
         30 |         60       25.00      100.00
------------+-----------------------------------
      Total |        240      100.00
```

(*Continued on next page*)

```
. meologit score dose2 dose3 dose4 age duration time || id2:

Mixed-effects ologit regression                 Number of obs     =        240
Group variable:             id2                 Number of groups  =         60

                                                Obs per group:
                                                              min =          4
                                                              avg =        4.0
                                                              max =          4

Integration method: mvaghermite                 Integration pts.  =          7

                                                Wald chi2(6)      =      78.05
Log likelihood = -221.62222                     Prob > chi2       =     0.0000

------------------------------------------------------------------------------
       score |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
       dose2 |  -.2077786    1.49336    -0.14   0.889    -3.134709    2.719152
       dose3 |  -1.227845   1.453444    -0.84   0.398    -4.076543    1.620854
       dose4 |  -1.802505   1.445967    -1.25   0.213    -4.636549    1.031539
         age |  -.0524312   .0346835    -1.51   0.131    -.1204095    .0155472
    duration |  -.0223472   .0147845    -1.51   0.131    -.0513242    .0066299
        time |    .235031   .0266534     8.82   0.000     .1827913    .2872707
-------------+----------------------------------------------------------------
       /cut1 |  -4.058966   2.130342                      -8.23436    .1164289
       /cut2 |  -1.285731   2.100772                     -5.403169    2.831706
       /cut3 |   1.416124   2.098766                     -2.697381     5.52963
-------------+----------------------------------------------------------------
id2          |
   var(_cons)|   13.36943   4.113677                      7.314834    24.43551
------------------------------------------------------------------------------
LR test vs. ologit model: chibar2(01) = 123.68        Prob >= chibar2 = 0.0000
```

3. *Compare the model from step 2 with a model including* `dosage` *as a continuous covariate instead of the dummy variables for dosage groups, using a likelihood ratio test at the 5% significance level.*

```
. estimates store model1

. meologit score dosage age duration time || id2:
Mixed-effects ologit regression                 Number of obs     =        240
Group variable:             id2                 Number of groups  =         60
                                                Obs per group:
                                                              min =          4
                                                              avg =        4.0
                                                              max =          4
Integration method: mvaghermite                 Integration pts.  =          7
                                                Wald chi2(4)      =      78.01
Log likelihood = -221.67293                     Prob > chi2       =     0.0000

       score        Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]

      dosage    -.1277912      .09199    -1.39   0.165    -.3080882    .0525058
         age    -.0558598    .0329918    -1.69   0.090    -.1205225    .0088029
    duration    -.0220667    .0147112    -1.50   0.134    -.0509001    .0067666
        time     .2349185    .0266471     8.82   0.000     .1826911    .2871459

       /cut1    -6.231995    2.860294                     -11.83807   -.6259222
       /cut2    -3.459762    2.816057                     -8.979132    2.059609
       /cut3    -.7598798    2.791079                     -6.230294    4.710534

id2
   var(_cons)    13.35747     4.11182                      7.306337    24.42019

LR test vs. ologit model: chibar2(01) = 123.61        Prob >= chibar2 = 0.0000
. estimates store model2
. lrtest model1 .
Likelihood-ratio test                                 LR chi2(2)  =      0.10
(Assumption: . nested in model1)                      Prob > chi2 =    0.9506
```

Linearity of the log-odds for the covariate `dosage` is not rejected at the 5% level ($L = 0.10$, df $= 2$, $p = 0.95$).

4. *Extend the model chosen in step 3 to include an interaction between* `dosage` *and* `time`*. Test the interaction using a Wald test at the 5% level of significance.*

```
. generate dosage_time = dosage*time

. meologit score dosage age duration time dosage_time || id2:
Mixed-effects ologit regression                 Number of obs     =        240
Group variable:             id2                 Number of groups  =         60
                                                Obs per group:
                                                              min =          4
                                                              avg =        4.0
                                                              max =          4
Integration method: mvaghermite                 Integration pts.  =          7
                                                Wald chi2(5)      =      77.50
Log likelihood = -221.49881                     Prob > chi2       =     0.0000
```

score	Coef.	Std. Err.	z	P>|z|	[95% Conf.	Interval]
dosage	-.1505731	.1008403	-1.49	0.135	-.3482165	.0470704
age	-.0556903	.0333188	-1.67	0.095	-.1209939	.0096133
duration	-.0222267	.0148576	-1.50	0.135	-.0513471	.0068938
time	.1981749	.0669428	2.96	0.003	.0669694	.3293803
dosage_time	.0017006	.0028829	0.59	0.555	-.0039497	.0073509
/cut1	-6.729309	3.012843			-12.63437	-.8242448
/cut2	-3.951823	2.966103			-9.765279	1.861632
/cut3	-1.23639	2.932581			-6.984144	4.511364
id2						
var(_cons)	13.62838	4.229277			7.418091	25.03782

```
LR test vs. ologit model: chibar2(01) = 123.87        Prob >= chibar2 = 0.0000
```

The dosage by time interaction is not significant at the 5% level ($z = 0.59$, $p = 0.56$).

5. *For the model selected in step 4, interpret the estimated* ORs *and random-intercept variance.*

```
. meologit score dosage age duration time || id2:, or

Mixed-effects ologit regression                 Number of obs     =        240
Group variable: id2                             Number of groups  =         60

                                                Obs per group:
                                                              min =          4
                                                              avg =        4.0
                                                              max =          4

Integration method: mvaghermite                 Integration pts.  =          7

                                                Wald chi2(4)      =      78.01
Log likelihood = -221.67293                     Prob > chi2       =     0.0000
```

score	Odds ratio	Std. err.	z	P>|z|	[95% conf.	interval]
dosage	.8800371	.0809546	-1.39	0.165	.7348505	1.053909
age	.9456717	.0311994	-1.69	0.090	.8864571	1.008842
duration	.978175	.0143901	-1.50	0.134	.9503736	1.00679
time	1.264806	.0337034	8.82	0.000	1.200444	1.332619
/cut1	-6.231995	2.860294			-11.83807	-.6259222
/cut2	-3.459762	2.816057			-8.979132	2.059609
/cut3	-.7598798	2.791079			-6.230294	4.710534
id2						
var(_cons)	13.35747	4.11182			7.306337	24.42019

```
Note: Estimates are transformed only in the first equation to odds ratios.
LR test vs. ologit model: chibar2(01) = 123.61        Prob >= chibar2 = 0.0000
```

Each extra gram of anesthetic per kilogram of weight is associated with an estimated 12% reduction in the odds of having a recovery score above a given cut-point, after controlling for covariates. This translates to a 72% ($-72 = 100(0.8800371^{10} - 1)$) reduction in the odds for a 10grams/kilogram increase. Each extra month of age is associated with an estimated 4% decrease in the odds of a high recovery score after controlling for the other covariates. For a one-year increase in age, the odds are estimated to decrease by 49% ($-20 = 100(0.9456717^{12} - 1)$). Each extra minute of surgery reduces the estimated odds of a high recovery score by 2%, corresponding to a 36% decrease ($-36 = 100(0.978175^{20} - 1)$) every 20 minutes. Finally, the estimated odds of a high recovery score increase over time after admission to the recovery room, by 26% per minute, after controlling for the other covariates.

The estimated random-intercept variance is large, giving an estimated residual intraclass correlation of the latent responses of 0.81 ($= 13.62838/(13.62838 + \pi^2/3)$).

6. ❖ *Extend the model selected in step 4 by relaxing the proportional-odds assumption for dosage (see section 11.2 on using the* `thresh()` *option in* `gllamm` *to relax proportional odds). Test whether the odds are proportional using a likelihood ratio test. To compare models fit by different commands, use the* `force` *option.*

   First store the estimates for the selected model:

```
. estimates store model2
```

   Then fit the model relaxing the proportional odds assumption in `gllamm`:

```
. eq thr: dosage
. gllamm score age duration time, i(id2)
>    link(ologit) thresh(thr) adapt
number of level 1 units = 240
number of level 2 units = 60

Condition Number = 919.91442

gllamm model

log likelihood = -217.9239
```

score	Coefficient	Std. err.	z	P>|z|	[95% conf.	interval]
score						
age	-.0591689	.0332862	-1.78	0.075	-.1244087	.0060709
duration	-.0222001	.0144823	-1.53	0.125	-.0505849	.0061847
time	.2428621	.0280665	8.65	0.000	.1878528	.2978715
_cut11						
dosage	.1970886	.1005634	1.96	0.050	-.000012	.3941892
_cons	-7.890947	3.003838	-2.63	0.009	-13.77836	-2.003532
_cut12						
dosage	.0501222	.0972455	0.52	0.606	-.1404754	.2407198
_cons	-1.732089	2.864999	-0.60	0.545	-7.347385	3.883207
_cut13						
dosage	.1317483	.1013735	1.30	0.194	-.0669401	.3304367
_cons	-.797635	2.891275	-0.28	0.783	-6.46443	4.86916

```
Variances and covariances of random effects
------------------------------------------------------------------------------

***level 2 (id2)

    var(1): 13.834367 (4.3019654)
------------------------------------------------------------------------------
```

Now perfom the LR test with the `force` option because different commands were used for the two models:

```
. estimates store model3

. lrtest model2 model3, force

Likelihood-ratio test                                 LR chi2(2)  =      7.50
(Assumption: model2 nested in model3)                 Prob > chi2 =    0.0235
```

We reject the proportional odds assumption for dosage group at the 5% level ($L = 7.50$, df $= 2$, $p = 0.02$).

7. *For age equal to 37 months, duration equal to 80 minutes, and time in recovery room equal to 15 minutes, produce a graph of predicted marginal probabilities similar to figure 11.13 for the model selected in step 6 or for the model selected in step 4. Also produce a corresponding stacked bar chart, treating dosage group as categorical.*

   First we set the explanatory variables equal to the required values:

```
. replace age=37
(232 real changes made)

. replace duration=80
(240 real changes made)

. replace time=15
(180 real changes made)
```

   Then we show how to make the graphs for the proportional odds model from step 4, which was estimated using `melogit` and stored as `model2`

```
. estimates restore model2
(results model2 are active now)
```

   Then we obtain the marginal probabilties and plot them:

```
. predict pr1-pr4, pr marginal
(using 7 quadrature points)

. graph bar (mean) pr1 pr2 pr3 pr4, over(dosage) stack
> legend(order(1 "Prob(y=1)" 2 "Prob(y=2)" 3 "Prob(y=3)" 4 "Prob(y=4)"))
```

   The graph is given in the left panel of figure 2. Note that the boundaries on the graph are not exactly parallel, but the logit transformation of the boundaries is. For the figure resembling figure 11.12, we need the cumulative probabilities that y is anything from 1 up to category s, for $s = 1, 2, 3, 4$:

```
. generate pr12 = pr1+pr2

. generate pr123 = pr12+pr3

. generate pr1234 = 1

. twoway (area pr1 dosage, sort fintensity(inten10))
> (rarea pr12 pr1 dosage, sort fintensity(inten50))
> (rarea pr123 pr12 dosage, sort fintensity(inten70))
> (rarea pr1234 pr123 dosage, sort fintensity(inten90)),
> legend(order(1 "Prob(y=1)" 2 "Prob(y=2)" 3 "Prob(y=3)" 4 "Prob(y=4)"))
> title("dosage")
```

   The graph is given in the left panel of figure 3.

   Now, we plot the same kinds of graphs for the model from step 6 which was estimated using `gllamm` and stored as `model3`. First, we delete the predicted probabilities for the proportional odds model so that we can use the same variable names again:

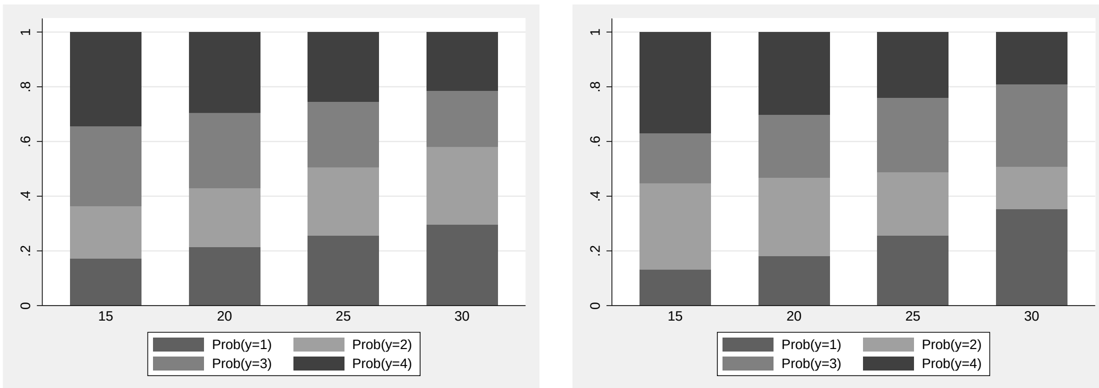

Figure 2: Area graphs of predicted marginal probabilities versus dosage groups, when age is 37 months, duration of surgery is 80 minutes, and recovery time is 15 minutes. Left panel is proportional odds model (model 2) and right panel relaxes proportional odds for dosage (model 3)

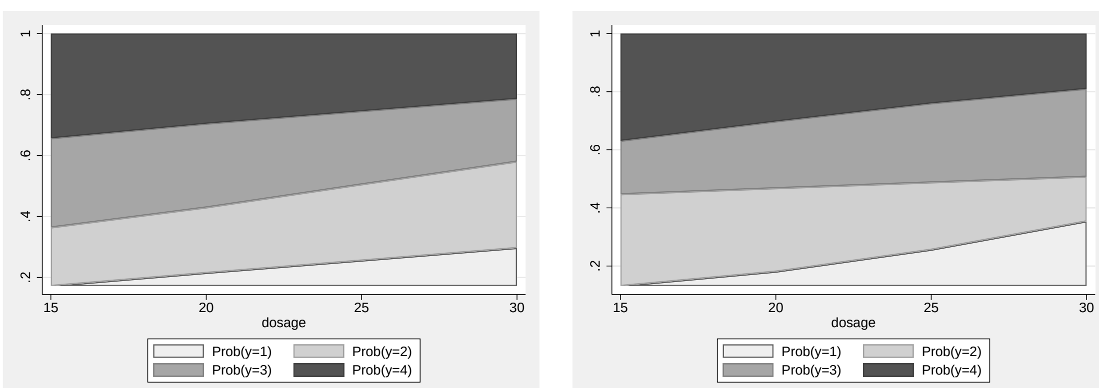

Figure 3: Stacked bar chart of predicted marginal probabilities for the dosage groups, when age is 37 months, duration of surgery is 80 minutes, and recovery time is 15 minutes. Left panel is proportional odds model (model 2) and right panel relaxes proportional odds for dosage (model 3)

```
. drop pr*
```

We can predict marginal cumulative probabilities using `gllapred`:

```
. estimates restore model3
(results model3 are active now)

. gllapred pr234, marg mu above(1) fsample
(mu will be stored in pr234)

. gllapred pr34, marg mu above(2) fsample
(mu will be stored in pr34)

. gllapred pr4, marg mu above(3) fsample
(mu will be stored in pr4)
```

These are converted to probabilities for each category as follows:

```
. generate pr1 = 1 - pr234

. generate pr2 = pr234 - pr34

. generate pr3 = pr34 - pr4
```

And the graph in the right panel of figure 2 is obtained using

```
. graph bar (mean) pr1 pr2 pr3 pr4, over(dosage) stack
> legend(order(1 "Prob(y=1)" 2 "Prob(y=2)" 3 "Prob(y=3)" 4 "Prob(y=4)"))
```

Again, for the figure resembling figure 11.12, we need the cumulative probabilities that y is anything from 1 up to category s, for $s = 1, 2, 3, 4$:

```
. generate pr12 = 1-pr34

. generate pr123 = 1-pr4

. generate pr1234 = 1

. twoway (area pr1 dosage, sort fintensity(inten10))
> (rarea pr12 pr1 dosage, sort fintensity(inten50))
> (rarea pr123 pr12 dosage, sort fintensity(inten70))
> (rarea pr1234 pr123 dosage, sort fintensity(inten90)),
> legend(order(1 "Prob(y=1)" 2 "Prob(y=2)" 3 "Prob(y=3)" 4 "Prob(y=4)"))
> xtitle("dosage")
```

The graph is given in the right panel of figure 3.

12.4 British-election data

1. *Create a variable,* `chosen`*, equal to 1 for the party voted for (*`rank` *equal to 1) and 0 for the other parties.*

```
. use elections, clear
. generate chosen = rank == 1
```

2. *Standardize* `lrdist` *and* `inflation` *to have mean 0 and variance 1. Produce all the dummy variables and interactions necessary to fit a conditional logistic regression model (using* `clogit`*) for* `chosen`*, with the following covariates: the standardized versions of* `lrdist` *and* `inflation`*, and the dummy varibles* `yr87`*,* `yr92`*,* `male`*, and* `manual`*. All variables except the standardized version of* `lrdist` *should have party-specific coefficients. There is no need for alternative-specific intercepts because interactions with both* `yr87` *and* `yr92` *are included.*

```
. egen inflat = std(inflation)
. egen dist = std(rldist)
. tabulate party, generate(p)
      party |      Freq.     Percent        Cum.
------------+-----------------------------------
          1 |      2,458       33.33       33.33
          2 |      2,458       33.33       66.67
          3 |      2,458       33.33      100.00
------------+-----------------------------------
      Total |      7,374      100.00
. rename p1 cons
. rename p2 lab
. rename p3 lib
. foreach var of varlist male inflat manual yr87 yr92 {
  2.         generate lab_`var' = lab*`var'
  3.         generate lib_`var' = lib*`var'
  4. }
```

3. *Fit the model using* `clogit` *and either* `gllamm` *or* `cmxtmixlogit`*, with Conservatives as the base outcome.*

```
. clogit chosen dist lab_*  lib_* , group(occ)
Conditional (fixed-effects) logistic regression   Number of obs   =       7374
                                                  LR chi2(11)     =    1434.69
                                                  Prob > chi2     =     0.0000
Log likelihood = -1983.0429                       Pseudo R2       =     0.2656

------------------------------------------------------------------------------
      chosen |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
        dist |  -1.134582   .0463711   -24.47   0.000    -1.225468   -1.043696
    lab_male |  -.7170468   .1247135    -5.75   0.000    -.9614808   -.4726129
  lab_inflat |     .40281   .0665768     6.05   0.000     .2723219     .533298
  lab_manual |   .5855308   .1298537     4.51   0.000     .3310223    .8400393
    lab_yr87 |  -.9940042   .1434858    -6.93   0.000    -1.275231   -.7127771
    lab_yr92 |  -.9786174   .1346003    -7.27   0.000    -1.242429   -.7148056
    lib_male |  -.6562548   .1194879    -5.49   0.000    -.8904468   -.4220627
  lib_inflat |   .3102374   .0623362     4.98   0.000     .1880607    .4324142
  lib_manual |  -.1422657   .1191864    -1.19   0.233    -.3758667    .0913353
    lib_yr87 |   -.785426   .1258898    -6.24   0.000    -1.032166   -.5386865
    lib_yr92 |  -1.068714   .1228379    -8.70   0.000    -1.309472   -.8279564
------------------------------------------------------------------------------
```

(*Continued on next page*)

Using `gllamm`:

```
. gllamm party dist lab_* lib_*, nocons i(occ) link(mlogit)
>  expanded(occ chosen o) init

number of level 1 units = 7374

Condition Number = 7.2688994

gllamm model

log likelihood = -1983.0429
```

party	Coef.	Std. Err.	z	P>|z|	[95% Conf.	Interval]
dist	-1.134582	.0463711	-24.47	0.000	-1.225468	-1.043696
lab_male	-.7170468	.1247135	-5.75	0.000	-.9614807	-.4726128
lab_inflat	.40281	.0665768	6.05	0.000	.272322	.5332981
lab_manual	.585531	.1298537	4.51	0.000	.3310225	.8400395
lab_yr87	-.9940045	.1434858	-6.93	0.000	-1.275232	-.7127774
lab_yr92	-.9786177	.1346003	-7.27	0.000	-1.24243	-.7148059
lib_male	-.6562546	.1194879	-5.49	0.000	-.8904466	-.4220626
lib_inflat	.3102375	.0623362	4.98	0.000	.1880608	.4324142
lib_manual	-.1422654	.1191864	-1.19	0.233	-.3758663	.0913356
lib_yr87	-.7854264	.1258898	-6.24	0.000	-1.032166	-.5386869
lib_yr92	-1.068715	.1228379	-8.70	0.000	-1.309473	-.8279569

(*Continued on next page*)

Using `cmxtmixlogit`:

```
. cmset serialno occ party
panel data: panels serialno and time occ
note: case identifier _caseid generated from serialno occ
note: panel by alternatives identifier _panelaltid generated from serialno party
                    caseid variable:  _caseid
              alternatives variable:  party
      panel by alternatives variable:  _panelaltid (unbalanced)
                      time variable:  occ, 1 to 2784
                              delta:  1 unit

note: data have been xtset

. cmxtmixlogit chosen dist lab_* lib_*, noconstant

Mixed logit choice model                      Number of obs       =      7,374
                                              Number of cases     =      2,458
Panel variable: serialno                      Number of panels    =      1,344

Time variable: occ                            Cases per panel: min =          1
                                                               avg =        1.9
                                                               max =          2

                                              Alts per case:   min =          3
                                                               avg =        3.0
                                                               max =          3

Integration points:              0               Wald chi2(11)    =     857.30
Log likelihood =        -1983.0429               Prob > chi2      =     0.0000

------------------------------------------------------------------------------
      chosen |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
party        |
        dist |  -1.134582   .0463711   -24.47   0.000    -1.225468   -1.043696
    lab_male |  -.7170469   .1247135    -5.75   0.000    -.9614808   -.4726129
  lab_inflat |     .40281   .0665768     6.05   0.000     .2723219     .533298
  lab_manual |   .5855308   .1298537     4.51   0.000     .3310223    .8400393
    lab_yr87 |  -.9940042   .1434858    -6.93   0.000    -1.275231   -.7127771
    lab_yr92 |  -.9786174   .1346003    -7.27   0.000    -1.242429   -.7148056
    lib_male |  -.6562548   .1194879    -5.49   0.000    -.8904468   -.4220627
  lib_inflat |   .3102374   .0623362     4.98   0.000     .1880607    .4324142
  lib_manual |  -.1422657   .1191864    -1.19   0.233    -.3758667    .0913353
    lib_yr87 |   -.785426   .1258898    -6.24   0.000    -1.032166   -.5386865
    lib_yr92 |  -1.068714   .1228379    -8.70   0.000    -1.309472   -.8279564
------------------------------------------------------------------------------
```

4. Extend the model to include a person-level random slope for `lrdist`, and fit the extended model in `gllamm` with 12-point adaptive quadrature or `cmxtmixlogit` with the default number of integration points (the latter will take considerably longer).

   We first fit the model in `gllamm`:

```
. eq slope: dist
. gllamm party dist lab_* lib_*, nocons i(serialno) eqs(slope)
>    link(mlogit) expanded(occ chosen o) adapt
number of level 1 units = 7374
number of level 2 units = 1344

Condition Number = 8.2098824

gllamm model

log likelihood = -1940.8731
```

party	Coef.	Std. Err.	z	P>|z|	[95% Conf.	Interval]
dist	-1.667974	.0950608	-17.55	0.000	-1.854289	-1.481658
lab_male	-.8029514	.1462798	-5.49	0.000	-1.089655	-.5162482
lab_inflat	.4829104	.0792102	6.10	0.000	.3276613	.6381595
lab_manual	.6980803	.1539755	4.53	0.000	.3962939	.9998668
lab_yr87	-1.09047	.1663477	-6.56	0.000	-1.416505	-.7644342
lab_yr92	-1.118557	.1568355	-7.13	0.000	-1.425949	-.8111654
lib_male	-.7209999	.1358186	-5.31	0.000	-.9871995	-.4548003
lib_inflat	.3926299	.0720597	5.45	0.000	.2513955	.5338644
lib_manual	-.0870119	.1367104	-0.64	0.524	-.3549594	.1809355
lib_yr87	-.8387357	.1429854	-5.87	0.000	-1.118982	-.5584894
lib_yr92	-1.177546	.1389775	-8.47	0.000	-1.449937	-.9051555

```
Variances and covariances of random effects
------------------------------------------------------------------------------

***level 2 (serialno)

    var(1): 1.0594742 (.21654741)
------------------------------------------------------------------------------
```

(*Continued on next page*)

Now we fit the same model in `cmxtmixlogit`:

```
. cmxtmixlogit chosen lab_* lib_*, noconstant random(dist)
Mixed logit choice model                      Number of obs       =      7,374
                                              Number of cases     =      2,458
Panel variable: serialno                      Number of panels    =      1,344
Time variable: occ                            Cases per panel: min =          1
                                                               avg =        1.9
                                                               max =          2
                                              Alts per case:   min =          3
                                                               avg =        3.0
                                                               max =          3
Integration sequence:      Hammersley
Integration points:               654              Wald chi2(11)  =     601.09
Log simulated likelihood = -1940.8473              Prob > chi2    =     0.0000

------------------------------------------------------------------------------
      chosen |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
party        |
    lab_male |  -.8029082   .1461815    -5.49   0.000    -1.089419   -.5163977
  lab_inflat |   .4827921   .0791922     6.10   0.000     .3275782     .638006
  lab_manual |   .6979731   .1538849     4.54   0.000     .3963642    .9995821
    lab_yr87 |  -1.090001   .1662027    -6.56   0.000    -1.415753   -.7642502
    lab_yr92 |  -1.118178   .1567131    -7.14   0.000     -1.42533   -.8110255
    lib_male |   -.720856   .1357229    -5.31   0.000     -.986868    -.454844
  lib_inflat |   .3924501   .0720178     5.45   0.000     .2512977    .5336024
  lib_manual |  -.0868981   .1366305    -0.64   0.525     -.354689    .1808927
    lib_yr87 |   -.838884   .1428815    -5.87   0.000    -1.118927   -.5588414
    lib_yr92 |  -1.177561   .1388967    -8.48   0.000    -1.449793   -.9053282
        dist |  -1.667532   .0946464   -17.62   0.000    -1.853036   -1.482029
-------------+----------------------------------------------------------------
/Normal      |
    sd(dist) |   1.026139   .1024326                      .8437935    1.247889
------------------------------------------------------------------------------
```

5. *Write down the model and interpret the estimates.*

The following model is specified for the conditional probability that party s is chosen by respondent j at occasion i, given the covariates and the random coefficient ζ_{2j} for `lrdist`:

$$\Pr(y_{ij}=s|x_{2ij}^{[s]},\mathbf{x}_{ij},\zeta_{2j})$$
$$= \frac{\exp\left\{(\beta_2+\zeta_{2j})x_{2ij}^{[s]} + \beta_3^{[s]}x_{3j} + \beta_4^{[s]}x_{4ij} + \beta_5^{[s]}x_{5j} + \beta_6^{[s]}x_{6i} + \beta_7^{[s]}x_{7i}\right\}}{\sum_{c=1}^{3}\exp\left\{(\beta_2+\zeta_{2j})x_{2ij}^{[c]} + \beta_3^{[c]}x_{3j} + \beta_4^{[c]}x_{4ij} + \beta_5^{[c]}x_{5j} + \beta_6^{[c]}x_{6i} + \beta_7^{[c]}x_{7i}\right\}}$$

Here $x_{2ij}^{[s]}$ represents `lrdist` for party s, x_{3j} represents `male`, x_{4ij} represents `inflation`, x_{5j} represents `manual`, x_{6i} represents `yr87`, and x_{7i} represents `yr92`. It is assumed that the random coefficient ζ_{2j} has a normal distribution with zero mean and variance ψ, and that the covariates are independent of the random coefficient.

We now turn to the interpretation of the estimates. Controlling for the other covariates, the conditional or respondent-specific odds of choosing a party decreases by 81% (-81% = 100% $\times$ $\exp(-1.668452)-1$) as the distance between the party and the respondent on the left-right political dimension increases by one unit. The variance of the respondent-specific effects $\beta_2+\zeta_{2j}$ is estimated as 1.0384731 so a 95% range of the odds ratio is $(\exp(-1.668452 - 1.96\sqrt{1.0384731}, \exp(-1.668452 - 1.96\sqrt{1.0384731}) = (0.03, 1.39)$.

The following interpretations are all in terms of conditional odds with Conservatives as base-category and given the other covariates.

We first consider the odds of choosing Labour. The odds of choosing Labour in 1987 is estimated as 0.34=$\exp(-1.088198)$ when all covariates are zero. The odds of choosing Labour in 1992 is estimated as 0.33=$\exp(-1.11707)$ when all covariates are zero. The odds of choosing Labour is estimated as 55% (-55% = 100% $(\exp(-0.8026911)-1)$) lower for males than for females. The odds of choosing Labour is estimated as 62% (62% = 100% $(\exp(0.4823476)-1)$) higher when the perceived inflation rating increases by one unit (which might be explained by the fact that Conservatives were the incumbents). The odds of choosing Labour is estimated as 100% (100% = 100% $(\exp(0.6978195)-1)$) higher for respondents whose father was a manual worker compared to the father not being a manual worker.

We then consider the odds of choosing Liberals. The odds of choosing Liberals in 1987 is estimated as 0.43=$\exp(-0.8391223)$ when all covariates are zero. The odds of choosing Liberals in 1992 is estimated as 0.31=$\exp(-1.177754)$ when all covariates are zero. The odds of choosing Liberals is estimated as 51% (-51% = 100% $(\exp(-0.720465)-1)$) lower for males than for females. The odds of choosing Liberals is estimated as 34% (34% = 100% $(\exp(0.2920127)-1)$) higher when the perceived inflation rating increases by one unit (which might be explained by the fact that Conservatives were the incumbents). The odds of choosing Liberals is estimated as 8% (-8% = 100% $(\exp(-0.0866056)-1)$) lower for respondents whose father was a manual worker compared to the father not being a manual worker.

6. *Instead of including a random slope for* `lrdist`*, include correlated person-level random intercepts for Labour and Liberal. Either use* `gllamm` *with 9-point adaptive quadrature or* `cmxtmixlogit` *with the default number of integration points (the latter will take considerably longer).* `cmxtmixlogit` *automatically includes fixed coefficients for Labour and Liberal, so the model is not identified unless you remove, for instance, the interactions between* `yr87` *and the Labour and Liberal dummy variables.*

   We start with `gllamm`:

```
. gllamm party dist lab_* lib_*, nocons i(serialno) nrf(2) eqs(lab lib)
>    link(mlogit) expanded(occ chosen o) nip(9) adapt

number of level 1 units = 7374
number of level 2 units = 1344

Condition Number = 10.937832

gllamm model

log likelihood = -1788.3248
```

party	Coef.	Std. Err.	z	P>|z|	[95% Conf.	Interval]
dist	-2.088836	.1409049	-14.82	0.000	-2.365004	-1.812667
lab_male	-1.250304	.315439	-3.96	0.000	-1.868553	-.6320545
lab_inflat	.7647105	.1418849	5.39	0.000	.4866213	1.0428
lab_manual	1.494767	.3402737	4.39	0.000	.827843	2.161692
lab_yr87	-2.001023	.3610284	-5.54	0.000	-2.708626	-1.29342
lab_yr92	-1.842304	.3375043	-5.46	0.000	-2.5038	-1.180808
lib_male	-1.129491	.3065094	-3.69	0.000	-1.730239	-.5287439
lib_inflat	.6435229	.1329496	4.84	0.000	.3829465	.9040992
lib_manual	.126338	.3187767	0.40	0.692	-.4984528	.7511288
lib_yr87	-1.584461	.3239303	-4.89	0.000	-2.219352	-.9495688
lib_yr92	-2.044765	.3265231	-6.26	0.000	-2.684738	-1.404791

```
Variances and covariances of random effects
------------------------------------------------------------------------------

***level 2 (serialno)

    var(1): 14.318181 (3.0102436)
    cov(2,1): 11.308938 (2.4086857) cor(2,1): .79114588

    var(2): 14.270613 (2.3786588)
------------------------------------------------------------------------------
```

(*Continued on next page*)

Now we fit the same model with `cmxtmixlogit`:

```
. cmxtmixlogit chosen dist lab_male lab_inflat lab_manual lab_yr92 lib_male
>    lib_inflat lib_manual lib_yr92, noconstant random(lab lib, correlated)

Mixed logit choice model                      Number of obs       =      7,374
                                              Number of cases     =      2,458
Panel variable: serialno                      Number of panels    =      1,344

Time variable: occ                            Cases per panel: min =          1
                                                               avg =        1.9
                                                               max =          2

                                              Alts per case:   min =          3
                                                               avg =        3.0
                                                               max =          3
Integration sequence:      Hammersley
Integration points:               704              Wald chi2(11)   =     319.07
Log simulated likelihood = -1788.6669              Prob > chi2     =     0.0000
```

chosen	Coef.	Std. Err.	z	P>|z|	[95% Conf.	Interval]
party						
dist	-2.075754	.1378129	-15.06	0.000	-2.345863	-1.805646
lab_male	-1.237107	.3129861	-3.95	0.000	-1.850549	-.6236655
lab_inflat	.7558056	.1404104	5.38	0.000	.4806063	1.031005
lab_manual	1.476726	.3375285	4.38	0.000	.8151818	2.138269
lab_yr92	.1538816	.2122281	0.73	0.468	-.2620779	.5698411
lib_male	-1.119151	.304987	-3.67	0.000	-1.716915	-.5213878
lib_inflat	.6365243	.1317774	4.83	0.000	.3782455	.8948032
lib_manual	.1154523	.3173066	0.36	0.716	-.5064572	.7373618
lib_yr92	-.4634786	.200367	-2.31	0.021	-.8561907	-.0707665
lab	-1.98754	.3588781	-5.54	0.000	-2.690928	-1.284152
lib	-1.587364	.3204624	-4.95	0.000	-2.215459	-.9592695
/Normal						
sd(lab)	3.697475	.3581382			3.058143	4.470467
corr(lab,lib)	.783795	.0501663	15.62	0.000	.6641424	.8642963
sd(lib)	3.706856	.2962614			3.169388	4.335467

Here the coefficient of `lab` corresponds to `lab_yr87` in the previous model and the sum of the coefficients of `lab` and `lab_yr92` corresponds to `lab_yr92` in the previous model (similarly for the Liberal party). We can use `lincom` to translate between the different parameterizations:

```
. lincom lab+lab_yr92
 ( 1)  [party]lab_yr92 + [party]lab = 0
```

chosen	Coef.	Std. Err.	z	P>|z|	[95% Conf.	Interval]
(1)	-1.833658	.335719	-5.46	0.000	-2.491655	-1.175661

```
. lincom lib+lib_yr92
 ( 1)  [party]lib_yr92 + [party]lib = 0
```

chosen	Coef.	Std. Err.	z	P>|z|	[95% Conf.	Interval]
(1)	-2.050843	.3225191	-6.36	0.000	-2.682969	-1.418717

The `gllamm` and `cmxtmixlogit` estimates are not identical to two decimal places. Try using `gllamm` with more quadrature points to make the estimates closer to those from `cmxtmixlogit`.

13.1 Epileptic-fit data

1. *Model II in Breslow and Clayton is a log-linear (Poisson regression) model with covariates* `lbas`, `treat`, `lbas_trt`, `lage`, *and* `v4`, *and a normally distributed random intercept for subjects. Fit this model using* `mepoisson`

```
. use epilep, clear
. mepoisson y lbas treat lbas_trt lage v4 || subj:
Mixed-effects Poisson regression                Number of obs     =        236
Group variable:            subj                 Number of groups  =         59
                                                Obs per group:
                                                              min =          4
                                                              avg =        4.0
                                                              max =          4
Integration method: mvaghermite                 Integration pts.  =          7
                                                Wald chi2(5)      =     121.70
Log likelihood = -665.29067                     Prob > chi2       =     0.0000

           y        Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]

        lbas     .8844225   .1312033     6.74   0.000     .6272689    1.141576
       treat    -.9330306   .4007512    -2.33   0.020    -1.718489   -.1475727
    lbas_trt     .3382561   .2033021     1.66   0.096    -.0602087     .736721
        lage     .4842226   .3471905     1.39   0.163    -.1962582    1.164703
          v4    -.1610871   .0545758    -2.95   0.003    -.2680536   -.0541206
       _cons     2.114306   .2196676     9.63   0.000     1.683766    2.544847

subj
   var(_cons)    .2528664   .0589844                      .1600801     .399434

LR test vs. Poisson model: chibar2(01) = 304.74       Prob >= chibar2 = 0.0000
```

The corresponding `gllamm` command is

```
gllamm y lbas treat lbas_trt lage v4, i(subj) link(log) family(poisson) adapt
```

2. *Breslow and Clayton also considered a random-coefficient model (Model IV) using the variable* `visit` *instead of* `v4`*. The effect of* `visit` z_{ij} *varies randomly between subjects. The model can be written as*

$$\log(\mu_{ij}) \;=\; \beta_1 + \beta_2 x_{2j} + \cdots + \beta_5 x_{5j} + \beta_6 z_{ij} + \zeta_{1j} + \zeta_{2j} z_{ij}$$

where the subject-specific random intercept ζ_{1j} *and slope* ζ_{2j} *have a bivariate normal distribution, given the covariates. Fit this model using* `mepoisson` *or* `gllamm` *(the latter is required for step 3 of this exercise).*

The `mepoisson` command produces the following output (corresponding output from `gllamm` is given under step 3):

```
. mepoisson y lbas treat lbas_trt lage visit || subj: visit, covariance(unstructured)

Mixed-effects Poisson regression                Number of obs     =        236
Group variable:            subj                 Number of groups  =         59

                                                Obs per group:
                                                              min =          4
                                                              avg =        4.0
                                                              max =          4

Integration method: mvaghermite                 Integration pts.  =          7

                                                Wald chi2(5)      =     115.60
Log likelihood = -655.68095                     Prob > chi2       =     0.0000

------------------------------------------------------------------------------
           y |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
        lbas |   .8849666   .1312244     6.74   0.000     .6277715    1.142162
       treat |  -.9286531   .4020804    -2.31   0.021    -1.716716   -.1405901
    lbas_trt |   .3379723   .2044067     1.65   0.098    -.0626574     .738602
        lage |   .4767056    .353527     1.35   0.178    -.2161946    1.169606
       visit |  -.2664103   .1646967    -1.62   0.106    -.5892098    .0563893
       _cons |   2.099559   .2203214     9.53   0.000     1.667737    2.531381
-------------+----------------------------------------------------------------
subj         |
  var(visit) |   .5314793    .229384                      .2280929      1.2384
  var(_cons) |   .2515327   .0588175                      .1590569    .3977741
-------------+----------------------------------------------------------------
subj         |
 cov(visit,_cons)|  .002872   .0886268     0.03   0.974    -.1708334    .1765774
------------------------------------------------------------------------------
LR test vs. Poisson model: chi2(3) = 324.54               Prob > chi2 = 0.0000

Note: LR test is conservative and provided only for reference.
```

3. ❖ *Plot the posterior mean counts versus time for 12 patients in each treatment group based on the model from Step 2. This requires fitting the model from step 2 in* `gllamm`.

We first fit the model in the previous step using `gllamm` (if you used `mepoisson` there):

```
. eq int: cons

. eq slope: visit

. gllamm y lbas treat lbas_trt lage visit, i(subj)
>   link(log) family(poisson) nrf(2) eqs(int slope)
>   nip(15) adapt
number of level 1 units = 236
number of level 2 units = 59

Condition Number = 9.3160203

gllamm model

log likelihood = -655.68102
```

y	Coef.	Std. Err.	z	P>|z|	[95% Conf.	Interval]
lbas	.8849767	.1312521	6.74	0.000	.6277273	1.142226
treat	-.9286588	.4021646	-2.31	0.021	-1.716887	-.1404307
lbas_trt	.3379757	.2044446	1.65	0.098	-.0627284	.7386798
lage	.4767191	.3536223	1.35	0.178	-.216368	1.169806
visit	-.2664098	.1647096	-1.62	0.106	-.5892346	.056415
_cons	2.099555	.2203713	9.53	0.000	1.667635	2.531474

```
Variances and covariances of random effects
------------------------------------------------------------------------------

***level 2 (subj)

    var(1): .25149332 (.05878944)
    cov(2,1): .00287152 (.08870194) cor(2,1): .00785426

    var(2): .53148073 (.22938513)
------------------------------------------------------------------------------
```

We can then obtain posterior mean counts by using `gllapred`:

```
. gllapred pred, mu
(mu will be stored in pred)
Non-adaptive log-likelihood: -659.19989
 -658.7592  -656.0947  -655.6810  -655.6810  -655.6810
log-likelihood:-655.68102

. sort treat subj

. by treat subj: generate f=_n==1

. by treat: generate id=sum(f)

. twoway line pred visit if id<13 & treat==0, by(id)

. twoway line pred visit if id<13 & treat==1, by(id)
```

The graphs are shown in figures 4 and 5.

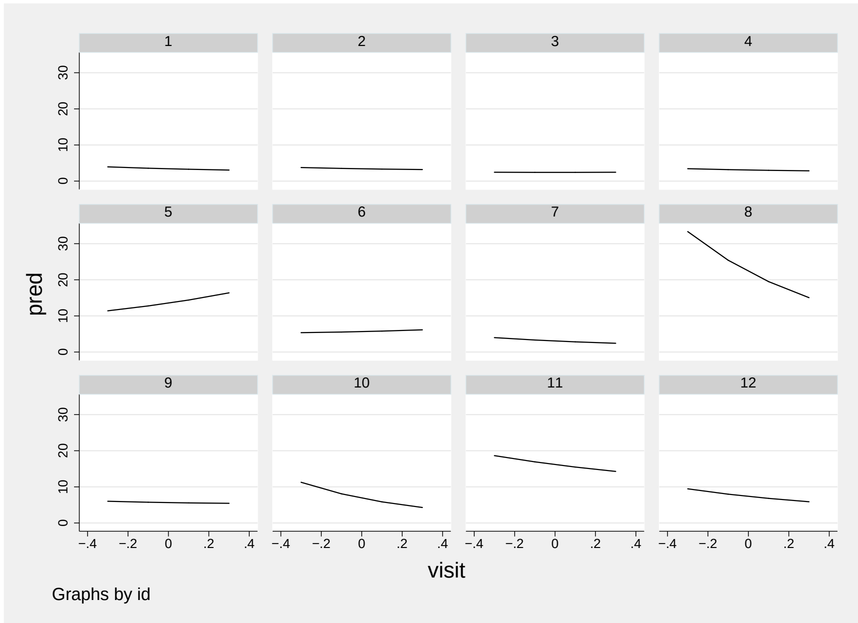

Figure 4: Posterior mean number of epileptic fits versus time for placebo group

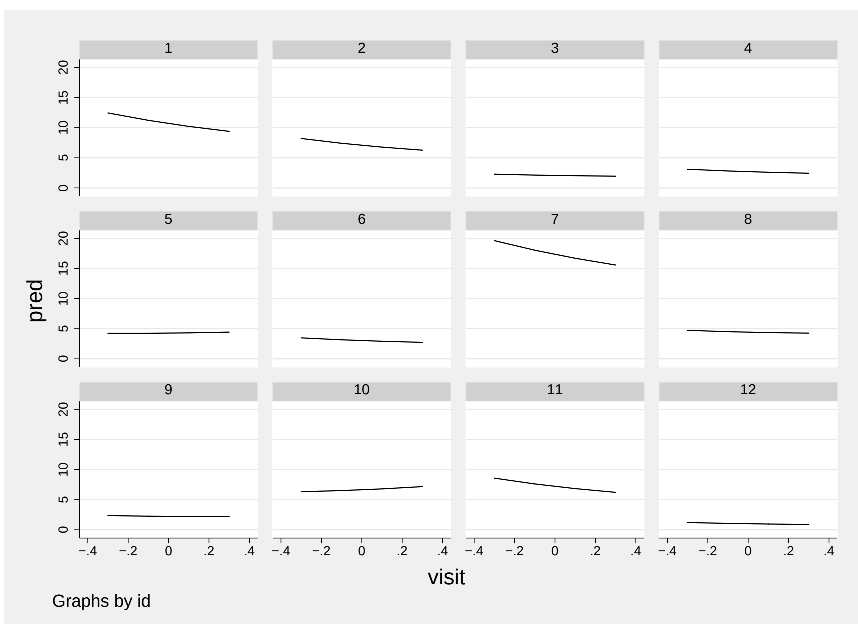

Figure 5: Posterior mean number of epileptic fits versus time for treatment group

14.7 Cigarette data

1. *Expand the data to person–period data.*

```
. use cigarette, clear
. generate id=_n
. expand time
(1670 observations created)
. by id, sort: gen t = _n
. generate y=0
. by id (t), sort: replace y = event if _n==_N
(634 real changes made)
```

2. *Estimate the discrete-time model that assumes the continuous-time hazards to be proportional. Include* `cc`, `tv`, *and their interaction as explanatory variables and specify a random intercept for classes. Use dummy variables for periods.*

```
. tabulate t, generate(occ)
```

t	Freq.	Percent	Cum.
1	1,556	48.23	48.23
2	1,082	33.54	81.77
3	588	18.23	100.00
Total	3,226	100.00	

```
. mecloglog y male cc tv cc_tv occ2 occ3 || class:
Mixed-effects cloglog regression                Number of obs     =      3,226
Group variable:           class                 Number of groups  =        134
                                                Obs per group:
                                                              min =          3
                                                              avg =       24.1
                                                              max =         54
Integration method: mvaghermite                 Integration pts.  =          7
                                                Wald chi2(6)      =      12.09
Log likelihood = -1592.3537                     Prob > chi2       =     0.0599
```

y	Coef.	Std. Err.	z	P>|z|	[95% Conf.	Interval]
male	.0594819	.0804729	0.74	0.460	-.0982421	.2172058
cc	.129357	.1216004	1.06	0.287	-.1089754	.3676895
tv	.0914655	.1222319	0.75	0.454	-.1481046	.3310356
cc_tv	-.1605053	.1747716	-0.92	0.358	-.5030515	.1820408
occ2	.0462722	.0918315	0.50	0.614	-.1337142	.2262586
occ3	.32482	.1042103	3.12	0.002	.1205716	.5290685
_cons	-1.707058	.1068043	-15.98	0.000	-1.91639	-1.497725
class						
var(_cons)	.0348174	.0300665			.0064083	.1891694

```
LR test vs. cloglog model: chibar2(01) = 1.76         Prob >= chibar2 = 0.0924
```

3. *Interpret the exponentials of the estimated regression coefficients.*

```
. mecloglog, eform

Mixed-effects cloglog regression                Number of obs     =      3,226
Group variable:           class                 Number of groups  =        134

                                                Obs per group:
                                                              min =          3
                                                              avg =       24.1
                                                              max =         54

Integration method: mvaghermite                 Integration pts.  =          7

                                                Wald chi2(6)      =      12.09
Log likelihood = -1592.3537                     Prob > chi2       =     0.0599

------------------------------------------------------------------------------
           y |     exp(b)   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
        male |   1.061287   .0854048     0.74   0.460     .9064294      1.2426
          cc |   1.138096    .138393     1.06   0.287     .8967524    1.444393
          tv |   1.095779   .1339392     0.75   0.454     .8623409    1.392409
       cc_tv |   .8517133   .1488553    -0.92   0.358     .6046827    1.199663
        occ2 |   1.047359   .0961806     0.50   0.614       .87484      1.2539
        occ3 |   1.383782   .1442043     3.12   0.002     1.128141     1.69735
       _cons |   .1813988   .0193742   -15.98   0.000     .1471371    .2236384
-------------+----------------------------------------------------------------
class        |
   var(_cons)|   .0348174   .0300665                      .0064083    .1891694
------------------------------------------------------------------------------
Note: Estimates are transformed only in the first equation.
LR test vs. cloglog model: chibar2(01) = 1.76         Prob >= chibar2 = 0.0924
```

At the 5% level of significance there is not sufficient evidence to conclude that the interventions had any effects.

Specifically, for each intervention on its own (when the other intervention is not used), the hazard ratio does not differ significantly from 1. When combined with the other intervention, the hazard ratio for each intervention decreases by an estimated 15% (since the hazard ratio for the interaction is 0.85).

The hazards of smoking are estimated as 38% greater in 9th grade than in 7th grade after controlling for the other variables.

4. *Obtain the estimated residual intraclass correlation of the latent responses.*

You can calculate the estimated intraclass correlation using

```
. display .0348174/(.0348174+_pi^2/6)
.02072771
```

This is a very small correlation, and we also see from the last line of the `mecloglog` output that we cannot reject the null hypothesis (at the 5% level) that the true intraclass correlation is 0.

15.4 Bladder-cancer data

1. *Wei, Lin, and Weissfeld (1989) specify a marginal Cox regression model based on total time and semirestricted risk sets, where the risk set for a kth event includes risk intervals for all previous events ($< k$). They specify event-specific baseline hazards and allow the effects of* `treat`, `number`, *and* `size` *to differ between events. Fit this model.*

```
. use bladder, clear

. egen obs = group(enum id)

. stset stop, failure(event=1) id(obs)

                id:  obs
     failure event:  event == 1
obs. time interval:  (stop[_n-1], stop]
 exit on or before:  failure

------------------------------------------------------------------------------
        340  total observations
          0  exclusions
------------------------------------------------------------------------------
        340  observations remaining, representing
        340  subjects
        112  failures in single-failure-per-subject data
      8,522  total analysis time at risk and under observation
                                                at risk from t =         0
                                     earliest observed entry t =         0
                                          last observed exit t =        59

. sort id enum

. list id enum start stop event _t0 _t _d _st if id>6&id<10 & _st==1, sepby(id)
```

	id	enum	start	stop	event	_t0	_t	_d	_st
25.	7	1	0	18	0	0	18	0	1
26.	7	2	18	18	0	0	18	0	1
27.	7	3	18	18	0	0	18	0	1
28.	7	4	18	18	0	0	18	0	1
29.	8	1	0	5	1	0	5	1	1
30.	8	2	5	18	0	0	18	0	1
31.	8	3	18	18	0	0	18	0	1
32.	8	4	18	18	0	0	18	0	1
33.	9	1	0	12	1	0	12	1	1
34.	9	2	12	16	1	0	16	1	1
35.	9	3	16	18	0	0	18	0	1
36.	9	4	18	18	0	0	18	0	1

The model could be parameterized by having a coefficient for `treat`, `number`, and `size`, as well as coefficients for interactions of each of these variables with dummy variables for the second, third and fourth events. Instead, we will include interactions between dummy variables for *each event*, including the first, and `treat`, `number`, and `size`. We must then omit "main effects" for `treat`, `number`, and `size`:

(*Continued on next page*)

```
. stcox ibn.enum#(c.treat c.number c.size), strata(enum) vce(cluster id) efron

         failure _d:  event == 1
   analysis time _t:  stop
                 id:  obs

Stratified Cox regr. -- Efron method for ties

No. of subjects      =          340            Number of obs    =         340
No. of failures      =          112
Time at risk         =         8522
                                               Wald chi2(12)    =       34.32
Log pseudolikelihood =   -423.73286            Prob > chi2      =      0.0006

                                   (Std. Err. adjusted for 85 clusters in id)

------------------------------------------------------------------------------
             |               Robust
          _t | Haz. Ratio   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
 enum#c.treat |
           1 |   .5909733   .1874038    -1.66   0.097     .3174264    1.100253
           2 |   .5313625   .1968685    -1.71   0.088     .2570531    1.098396
           3 |   .4973349   .2103116    -1.65   0.099     .2171177    1.139207
           4 |   .5297029   .2649767    -1.27   0.204     .1987149    1.411999
             |
enum#c.number |
           1 |   1.268937   .0952058     3.17   0.002     1.095409    1.469955
           2 |   1.146744   .1012115     1.55   0.121     .9645825    1.363306
           3 |    1.18947   .1264058     1.63   0.103     .9658189    1.464911
           4 |   1.394411   .1621041     2.86   0.004      1.11029    1.751238
             |
  enum#c.size |
           1 |   1.072094   .0955849     0.78   0.435      .900206    1.276802
           2 |   .9251941   .1106043    -0.65   0.515     .7319378    1.169477
           3 |   .8074792   .1409972    -1.22   0.221     .5734553    1.137007
           4 |   .8134582   .1585875    -1.06   0.290     .5551233    1.192013
------------------------------------------------------------------------------
                                                             Stratified by enum
```

2. *Use* `testparm` *to test whether the coefficients of* `treat` *differ significantly between events (at the 5% level) and similarly for* `number` *and* `size`*.*

   In order to use `testparm`, it is better to use the more standard way of including interactions, where the dummy variable for event 1 is excluded and `treat`, `number`, and `size` are included:

(*Continued on next page*)

```
. stcox i.enum#(c.treat c.number c.size) c.treat c.number c.size,
>    strata(enum) vce(cluster id) efron

         failure _d:  event == 1
   analysis time _t:  stop
                 id:  obs

Stratified Cox regr. -- Efron method for ties

No. of subjects      =          340          Number of obs    =         340
No. of failures      =          112
Time at risk         =         8522
                                             Wald chi2(12)    =       34.32
Log pseudolikelihood =   -423.73286          Prob > chi2      =      0.0006

                                    (Std. Err. adjusted for 85 clusters in id)
```

_t	Haz. Ratio	Robust Std. Err.	z	P>|z|	[95% Conf.	Interval]
enum#c.treat						
2	.899131	.3020539	-0.32	0.752	.4654473	1.736903
3	.8415522	.337499	-0.43	0.667	.3834531	1.846928
4	.8963229	.4565585	-0.21	0.830	.3302854	2.432426
enum#c.number						
2	.9037042	.1068984	-0.86	0.392	.7167016	1.1395
3	.9373751	.11348	-0.53	0.593	.7393767	1.188396
4	1.098881	.1323528	0.78	0.434	.8678191	1.391464
enum#c.size						
2	.8629789	.0990377	-1.28	0.199	.6891505	1.080653
3	.7531798	.1153141	-1.85	0.064	.5579266	1.016764
4	.7587567	.1442884	-1.45	0.147	.5226783	1.101465
treat	.5909733	.1874038	-1.66	0.097	.3174264	1.100253
number	1.268937	.0952058	3.17	0.002	1.095409	1.469955
size	1.072094	.0955849	0.78	0.435	.900206	1.276802

```
                                                          Stratified by enum
. testparm enum#c.treat

 ( 1)  2.enum#c.treat = 0
 ( 2)  3.enum#c.treat = 0
 ( 3)  4.enum#c.treat = 0

           chi2(  3) =    0.24
         Prob > chi2 =    0.9715

. testparm enum#c.number

 ( 1)  2.enum#c.number = 0
 ( 2)  3.enum#c.number = 0
 ( 3)  4.enum#c.number = 0

           chi2(  3) =    5.86
         Prob > chi2 =    0.1186

. testparm enum#c.size

 ( 1)  2.enum#c.size = 0
 ( 2)  3.enum#c.size = 0
 ( 3)  4.enum#c.size = 0

           chi2(  3) =    3.61
         Prob > chi2 =    0.3065
```

None of the interactions are significant at the 5% level

3. *Fit the model by Wei, Lin, and Weissfeld (1989) but constraining all coefficients to be the same across events.*

```
. stcox treat number size, strata(enum) vce(cluster id) efron
         failure _d:  event == 1
   analysis time _t:  stop
                 id:  obs

Stratified Cox regr. -- Efron method for ties
No. of subjects      =          340            Number of obs    =         340
No. of failures      =          112
Time at risk         =         8522
                                               Wald chi2(3)     =       15.35
Log pseudolikelihood =   -426.14683            Prob > chi2      =      0.0015
                                  (Std. Err. adjusted for 85 clusters in id)
```

_t	Haz. Ratio	Robust Std. Err.	z	P>|z|	[95% Conf.	Interval]
treat	.5572209	.1726125	-1.89	0.059	.3036319	1.022604
number	1.23404	.0827266	3.14	0.002	1.0821	1.407316
size	.9496925	.0903613	-0.54	0.587	.788121	1.144388

```
                                                          Stratified by enum
```

4. *In their model (2), Prentice, Williams, and Peterson (1981) use counting process risk intervals with restricted risk sets and event-specific baseline hazards. Fit this model, assuming that* `treat`, `number`, *and* `size` *have the same coefficients across events.*

```
. stset stop, enter(start) failure(event=1) id(obs)
                id:  obs
     failure event:  event == 1
obs. time interval:  (stop[_n-1], stop]
 enter on or after:  time start
 exit on or before:  failure

------------------------------------------------------------------------------
      340  total obs.
      162  obs. end on or before enter()
------------------------------------------------------------------------------
      178  obs. remaining, representing
      178  subjects
      112  failures in single failure-per-subject data
     2480  total analysis time at risk, at risk from t =         0
                             earliest observed entry t =         0
                                  last observed exit t =        59

. sort id enum

. list id enum start stop event _t0 _t _d _st if id>6&id<10 & _st==1, sepby(id)
```

	id	enum	start	stop	event	_t0	_t	_d	_st
25.	7	1	0	18	0	0	18	0	1
29.	8	1	0	5	1	0	5	1	1
30.	8	2	5	18	0	5	18	0	1
33.	9	1	0	12	1	0	12	1	1
34.	9	2	12	16	1	12	16	1	1
35.	9	3	16	18	0	16	18	0	1

```
. stcox treat number size, strata(enum) vce(cluster id) efron

         failure _d:  event == 1
   analysis time _t:  stop
  enter on or after:  time start
                 id:  obs

Stratified Cox regr. -- Efron method for ties

No. of subjects      =          178             Number of obs   =        178
No. of failures      =          112
Time at risk         =         2480
                                                Wald chi2(3)    =       7.17
Log pseudolikelihood =   -315.99082             Prob > chi2     =     0.0665

                                  (Std. Err. adjusted for 85 clusters in id)
```

_t	Haz. Ratio	Robust Std. Err.	z	P>|z|	[95% Conf.	Interval]
treat	.71642	.147584	-1.62	0.105	.4784299	1.072796
number	1.127065	.0582599	2.31	0.021	1.018472	1.247238
size	.9915413	.0614766	-0.14	0.891	.8780828	1.11966

```
                                                          Stratified by enum
```

5. *Andersen and Gill (1982) also use counting process risk intervals, but they use unrestricted risk sets and assume that all events have a common baseline hazard function. Fit this model, again assuming that* `treat`*,* `number`*, and* `size` *have the same coefficients across events.*

```
. stcox c.treat c.number c.size, vce(cluster id) efron

         failure _d:  event == 1
   analysis time _t:  stop
  enter on or after:  time start
                 id:  obs

Cox regression -- Efron method for ties

No. of subjects      =          178             Number of obs   =        178
No. of failures      =          112
Time at risk         =         2480
                                                Wald chi2(3)    =      11.41
Log pseudolikelihood =   -449.98064             Prob > chi2     =     0.0097

                                  (Std. Err. adjusted for 85 clusters in id)
```

_t	Haz. Ratio	Robust Std. Err.	z	P>|z|	[95% Conf.	Interval]
treat	.6283318	.1678506	-1.74	0.082	.3722217	1.06066
number	1.191199	.0755395	2.76	0.006	1.051976	1.348848
size	.9572791	.0747412	-0.56	0.576	.821447	1.115572

6. *In their model (3), Prentice, Williams, and Peterson (1981) use gap time with restricted risk sets and event-specific baseline hazards. Fit this model, assuming that* `treat`, `number`, *and* `size` *have the same coefficients across events.*

```
. stset stop, origin(start) failure(event=1) id(obs)
                id:  obs
     failure event:  event == 1
obs. time interval:  (stop[_n-1], stop]
 exit on or before:  failure
    t for analysis:  (time-origin)
            origin:  time start

------------------------------------------------------------------------------
      340  total obs.
      162  obs. end on or before enter()
------------------------------------------------------------------------------
      178  obs. remaining, representing
      178  subjects
      112  failures in single failure-per-subject data
     2480  total analysis time at risk, at risk from t =         0
                             earliest observed entry t =         0
                                  last observed exit t =        59

. sort id enum

. list id enum start stop event _t0 _t _d _st if id>6&id<10 & _st==1, sepby(id)

     +-------------------------------------------------------+
     | id   enum   start   stop   event   _t0   _t   _d   _st |
     |-------------------------------------------------------|
 25. |  7      1       0     18       0     0   18    0     1 |
     |-------------------------------------------------------|
 29. |  8      1       0      5       1     0    5    1     1 |
 30. |  8      2       5     18       0     0   13    0     1 |
     |-------------------------------------------------------|
 33. |  9      1       0     12       1     0   12    1     1 |
 34. |  9      2      12     16       1     0    4    1     1 |
 35. |  9      3      16     18       0     0    2    0     1 |
     +-------------------------------------------------------+

. stcox treat number size, strata(enum) vce(cluster id) efron
         failure _d:  event == 1
   analysis time _t:  (stop-origin)
             origin:  time start
                 id:  obs

Stratified Cox regr. -- Efron method for ties

No. of subjects      =          178                Number of obs   =       178
No. of failures      =          112
Time at risk         =         2480
                                                   Wald chi2(3)    =     11.70
Log pseudolikelihood =   -358.96849                Prob > chi2     =    0.0085

                                  (Std. Err. adjusted for 85 clusters in id)
------------------------------------------------------------------------------
             |               Robust
          _t | Haz. Ratio   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
       treat |   .7565365   .1640954    -1.29   0.198     .4945398    1.157333
      number |    1.17122   .0600157     3.08   0.002     1.059305    1.294958
        size |   1.007443    .065196     0.11   0.909     .8874327    1.143682
------------------------------------------------------------------------------
                                                             Stratified by enum
```

7. *Compare and interpret the treatment-effect estimates from steps 3 to 6.*

   The estimated hazard ratios are 0.56 for total time semi-restricted, 0.72 for counting process, restricted, 0.63 for counting process unrestricted, and 0.76 for gap times, restricted. Only the total time semi-restricted estimate is nearly significant at the 5% level. The estimates can be interpreted as a 54% reduction in the hazard (largest effect size estimate) down to a 24% reduction in the hazard (smallest effect size estimate), controlling for number and maximum size of initial tumors.

16.2 Tower-of-London data

1. *Fit the two-level random-intercept model (random intercept for persons):*

$$\text{logit}\{\Pr(y_{ijk}=1 \mid \mathbf{x}_{ijk}, \zeta_{jk}^{(2)})\} \;=\; \beta_0 + \beta_1 x_{ijk} + \beta_2 g_{2ijk} + \beta_3 g_{3ijk} + \zeta_{jk}^{(2)}$$

where g_{2ijk} *and* g_{3ijk} *are dummy variables for groups 2 and 3, respectively, and* $\zeta_{jk}^{(2)} \sim N(0, \psi^{(2)})$ *is independent of the covariates* $\mathbf{x}_{ijk}$*. Here and throughout the exercise,* `level` *is treated as continuous.*

```
. use towerl, clear

. tabulate group, generate(g)

      GROUP |      Freq.     Percent        Cum.
------------+-----------------------------------
          1 |        194       28.66       28.66
          2 |        294       43.43       72.08
          3 |        189       27.92      100.00
------------+-----------------------------------
      Total |        677      100.00

. rename g2 relatives

. rename g3 schizo

. melogit dtlm level relatives schizo || id:

Mixed-effects logistic regression               Number of obs     =        677
Group variable:              id                 Number of groups  =        226

                                                Obs per group:
                                                              min =          2
                                                              avg =        3.0
                                                              max =          3

Integration method: mvaghermite                 Integration pts.  =          7

                                                Wald chi2(3)      =      74.77
Log likelihood = -305.95965                     Prob > chi2       =     0.0000

------------------------------------------------------------------------------
        dtlm |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
       level |  -1.648715   .1932597    -8.53   0.000    -2.027497   -1.269933
   relatives |  -.1690655   .3342397    -0.51   0.613    -.8241632    .4860322
      schizo |   -1.02274   .3938351    -2.60   0.009    -1.794642   -.2508373
       _cons |  -1.482555   .2834946    -5.23   0.000    -2.038194   -.9269154
-------------+----------------------------------------------------------------
id           |
   var(_cons)|   1.674663   .6609173                      .7726717     3.62961
------------------------------------------------------------------------------
LR test vs. logistic model: chibar2(01) = 15.86       Prob >= chibar2 = 0.0000

. estimates store mod0
```

The syntax for `gllamm` is

```
gllamm dtlm level relatives schizo, i(id) link(logit) family(binomial) adapt
```

2. *Fit the three-level random-intercept model (random intercepts for subjects and families):*

$$\text{logit}\{\Pr(y_{ijk}=1 \mid \mathbf{x}_{ijk}, \zeta_{jk}^{(2)}, \zeta_k^{(3)})\} \;=\; \beta_0 + \beta_1 x_{ijk} + \beta_2 g_{2ijk} + \beta_3 g_{3ijk} + \zeta_{jk}^{(2)} + \zeta_k^{(3)}$$

where $\zeta_{jk}^{(2)} \sim N(0, \psi^{(2)})$ *is independent of* $\zeta_k^{(3)} \sim N(0, \psi^{(3)})$ *and both random effects are assumed independent of* $\mathbf{x}_{ijk}$.

```
. melogit dtlm level relatives schizo || famnum: || id:
Mixed-effects logistic regression               Number of obs     =        677

--------------------------------------------------------
                |     No. of       Observations per Group
 Group Variable |     Groups    Minimum    Average    Maximum
----------------+---------------------------------------
         famnum |        118          2        5.7         27
             id |        226          2        3.0          3
--------------------------------------------------------

Integration method: mvaghermite                 Integration pts.  =          7

                                                Wald chi2(3)      =      74.90
Log likelihood = -305.12041                     Prob > chi2       =     0.0000
------------------------------------------------------------------------------
        dtlm |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
       level |  -1.648505   .1932075    -8.53   0.000    -2.027185   -1.269826
   relatives |  -.2486841   .3544076    -0.70   0.483    -.9433102     .445942
      schizo |  -1.052306   .3999921    -2.63   0.009    -1.836276   -.2683357
       _cons |  -1.485863   .2848455    -5.22   0.000     -2.04415   -.9275762
-------------+----------------------------------------------------------------
famnum       |
   var(_cons)|   .5692105   .5215654                      .0944757    3.429459
-------------+----------------------------------------------------------------
famnum>id    |
   var(_cons)|   1.137917   .6854853                      .3494165    3.705762
------------------------------------------------------------------------------
LR test vs. logistic model: chi2(2) = 17.54               Prob > chi2 = 0.0002
Note: LR test is conservative and provided only for reference.
. estimates store mod1
```

Subjects with schizophrenia perform significantly worse than unrelated healthy control subjects, whereas the healthy relatives of the subjects with schizophrenia do perform significantly worse than unrelated healthy control subjects (at the 5% level). Performance declines as the level of difficulty increases. There is more variability between subjects within families than between families after controlling for covariates.

The syntax for `gllamm` is

```
gllamm dtlm level relatives schizo, i(id famnum) link(logit) family(binomial) adapt
```

3. *Compare the models in steps 1 and 2 by using a likelihood-ratio test, but retain the three-level model even if the null hypothesis is not rejected at the 5% level.*

```
. lrtest mod0 mod1
Likelihood-ratio test                                 LR chi2(1)  =      1.68
(Assumption: mod0 nested in mod1)                     Prob > chi2 =    0.1951
Note: The reported degrees of freedom assumes the null hypothesis is not on
      the boundary of the parameter space.  If this is not true, then the
      reported test is conservative.
```

Since the random intercepts at the different levels are uncorrelated, we can divide the naïve p-value by 2 (see display 8.1, page 397) to obtain the correct asymptotic p-value of 0.10.

4. *Include a group (controls, relatives, schizophrenics) by level of difficulty interaction in the three-level model. Test the interaction by using both a Wald test and a likelihood-ratio test.*

```
. generate lev_rel = level*relatives

. generate lev_sch = level*schizo

. melogit dtlm level relatives schizo lev_rel lev_sch
>  || famnum: || id:

Mixed-effects logistic regression               Number of obs     =        677

--------------------------------------------------------------
                |     No. of       Observations per Group
 Group Variable |     Groups    Minimum    Average    Maximum
----------------+---------------------------------------------
         famnum |        118          2        5.7         27
             id |        226          2        3.0          3
--------------------------------------------------------------

Integration method: mvaghermite                 Integration pts.  =          7

                                                Wald chi2(5)      =      72.08
Log likelihood = -301.88298                     Prob > chi2       =     0.0000
------------------------------------------------------------------------------
        dtlm |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
       level |  -1.180708   .2643882    -4.47   0.000      -1.6989    -.662517
   relatives |  -.4365397   .3705962    -1.18   0.239    -1.162895    .2898156
      schizo |  -1.611146   .5116061    -3.15   0.002    -2.613876   -.6084166
     lev_rel |  -.6126014   .3527997    -1.74   0.082    -1.304076    .0788733
     lev_sch |  -1.176491   .5209267    -2.26   0.024    -2.197489   -.1554935
       _cons |  -1.356806    .279781    -4.85   0.000    -1.905167   -.8084453
-------------+----------------------------------------------------------------
famnum       |
   var(_cons)|   .5378161   .4857528                      .0915868    3.158164
-------------+----------------------------------------------------------------
famnum>id    |
   var(_cons)|   1.208996   .6959634                      .3912255    3.736134
------------------------------------------------------------------------------
LR test vs. logistic model: chi2(2) = 17.83               Prob > chi2 = 0.0001

Note: LR test is conservative and provided only for reference.
```

We obtain a Wald test by using `testparm`

```
. testparm lev_rel lev_sch

 ( 1)  [dtlm]lev_rel = 0
 ( 2)  [dtlm]lev_sch = 0

           chi2(  2) =    6.08
         Prob > chi2 =    0.0478
```

The interaction is significant at the 5% level according to the Wald test ($w = 6.09$, df $= 2$, $p = 0.048$). The corresponding likelihood-ratio test can be obtained using `lrtest`

```
. lrtest mod1 .

Likelihood-ratio test                                 LR chi2(2)  =      6.47
(Assumption: mod1 nested in .)                        Prob > chi2 =    0.0393
```

The likelihood-ratio statistic is 6.47 with two degrees of freedom, giving a p-value of 0.04.

For schizophrenics, performance declines faster with increasing level of difficulty than for controls ($z = -2.26$, $p = 0.024$).

5. *For the model in step 4, obtain predicted marginal or population-averaged probabilities. Plot the probabilities against the levels of difficulty with different curves for the three groups.* To obtain predicted marginal or population-averaged probabilities we can use `predict` (after fitting with `melogit`)

```
. predict prob, pr marginal
(using 7 quadrature points)
```

or `gllapred` (after fitting with `gllamm`)

```
. gllapred prob, mu marg
(mu will be stored in prob)
```

The plot can now be obtained as

```
. twoway (line prob level if group==1, sort)
>   (line prob level if group==2, sort lpatt(longdash))
>   (line prob level if group==3, sort lpatt(shortdash)),
>   xtitle(Level of difficulty) ytitle(Probability)
>   legend(order(1 "Controls" 2 "Relatives" 3 "Schizophrenics") row(1))
>   xlabel(-1 "Low" 0 "Medium" 1 "High")
```

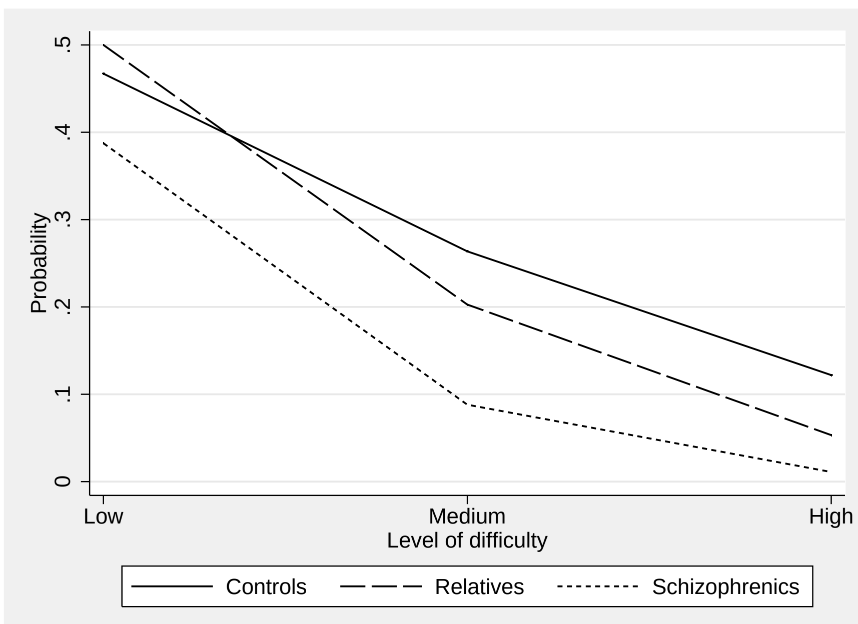

Figure 6: Predicted marginal probabilities as a function of level of difficulty for the three groups.