Interpreting and Visualizing Regression Models Using Stata

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Think back to the first time you learned about simple linear regression. You probably learned about the underlying theory of linear regression, the meaning of the regression coefficients, and how to create a graph of the regression line. The graph of the regression line provided a visual representation of the intercept and slope coefficients. Using such a graph, you could see that as the intercept increased, so did the overall height of the regression line, and as the slope increased, so did the tilt of the regression line. Within Stata, the `graph twoway lfit` command can be used to easily visualize the results of a simple linear regression.

Over time we learn about and use fancier and more abstract regression models—models that include covariates, polynomial terms, piecewise terms, categorical predictors, interactions, and nonlinear models such as logistic. Compared with a simple linear regression model, it can be challenging to visualize the results of such models. The utility of these fancier models diminishes if we have greater difficulty interpreting and visualizing the results.

With the introduction of the `marginsplot` command in Stata 12, visualizing the results of a regression model, even complex models, is a snap. As implied by the name, the `marginsplot` command works in tandem with the `margins` command by plotting (graphing) the results computed by the `margins` command. For example, after fitting a linear model, the `margins` command can be used to compute adjusted means as a function of one or more predictors. The `marginsplot` command graphs the adjusted means, allowing you to visually interpret the results.

The `margins` and `marginsplot` commands can be used following nearly all Stata estimation commands (including `regress`, `anova`, `logit`, `ologit`, and `mlogit`). Furthermore, these commands work with continuous linear predictors, categorical predictors, polynomial (power) terms, as well as interactions (for example, two-way interactions, three-way interactions). This book uses the `marginsplot` command not only as an interpretive tool, but also as an instructive tool to help you understand the results of regression models by visualizing them.

Categorical predictors pose special difficulties with respect to interpreting regression models, especially models that involve interactions of categorical predictors. Categorical predictors are traditionally coded using dummy (indicator) coding. Many research questions cannot be answered directly in terms of dummy variables. Furthermore, interactions involving dummy categorical variables can be confusing and even misleading. Stata 12 introduces the `contrast` command, a general-purpose command that can be
used to precisely test the effects of categorical variables by forming contrasts among
the levels of the categorical predictors. For example, you can compare adjacent groups,
compare each group with the overall mean, or compare each group with the mean of the
previous groups. The `contrast` command allows you to easily focus on the comparisons
that are of interest to you.

The `contrast` command works with interactions as well. You can test the simple
effect of one predictor at specific levels of another predictor or form interactions that
involve comparisons of your choosing. In the parlance of analysis of variance, you
can test simple effects, simple contrasts, partial interactions, and interaction contrasts.
These kinds of tests allow you to precisely understand and dissect interactions with
surgical precision. The `contrast` command works not only with the `regress` command,
but also with commands such as `logit`, `ologit`, `mlogit`, as well as random-effects
models like `xtmixed`.

As you can see, the scope of the application of the `margins`, `marginsplot`, and
`contrast` commands is broad. Likewise, so is the scope of this book. It covers con-
tinuous variables (modeled linearly, using polynomials, and piecewise), interactions of
continuous variables, categorical predictors, interactions of categorical predictors, as
well as interactions of continuous and categorical predictors. The book also illustrates
how the `margins`, `marginsplot`, and `contrast` commands can be used to interpret re-
sults from multilevel models, models where time is a continuous predictor, models with
time as a categorical predictor, nonlinear models (such as logistic regression or ordi-
nal logistic regression), and analyses that involve complex survey data. However, this
book does not contain information about the theory of these statistical models, how
to perform diagnostics for the models, the formulas for the models, and so forth. The
summary section concluding each chapter includes references to books and articles that
provide background for the techniques illustrated in the chapter.

My goal for this book is to provide simple and clear examples that illustrate how
to interpret and visualize the results of regression models. To that end, I have selected
examples that illustrate large effects generally combined with large sample sizes to create
patterns of effects that are easy to visualize. Most of the examples are based on real
data, but some are based on hypothetical data. In either case, I hope the examples help
you understand the results of your regression models so you can interpret and present
them with clarity and confidence.
(Pages omitted)
14 Continuous by categorical by categorical interactions

14.1 Chapter overview
This chapter considers models that involve the interaction of two categorical predictors with a linear continuous predictor. Such models blend ideas from chapter 10 on categorical by continuous interactions and ideas from chapter 8 on categorical by categorical interactions. As we saw in chapter 10, interactions of categorical and continuous predictors describe how the slope of the continuous variable differs as a function of the categorical variable. In chapter 8, we saw models that involve the interaction of two categorical variables. This chapter blends these two modeling techniques by exploring how the slope of the continuous variable varies as a function of the interaction of the two categorical variables.

Let’s consider a hypothetical example of a model with income as the outcome variable. The predictors include gender (a two-level categorical variable), education (treated as a three-level categorical variable), and age (a continuous variable). Income can be modeled as a function of each of the predictors, as well as the interactions of all the predictors. A three-way interaction of age by gender by education would imply that the effect of age interacts with gender by education. One way to visualize such an interaction would be to graph age on the x axis, with separate lines for the levels of education and separate graphs for gender. Figure 14.1 shows such an example, illustrating how the slope of the relationship between income and age varies as a function of education and gender.
Figure 14.1. Fitted values of income as a function of age, education, and gender

The graph can be augmented by a table that shows the age slope broken down by education and gender. Such a table is shown in 14.1. The age slope shown in each cell of table 14.1 reflects the slope of the relationship between income and age for each of the lines illustrated in figure 14.1. For example, $\beta_{3M}$ represents the age slope for male college graduates, and this slope is 1,300.

Table 14.1. The age slope by level of education and gender

<table>
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<th></th>
<th>Non-HS grad</th>
<th>HS grad</th>
<th>CO grad</th>
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<tr>
<td>Male</td>
<td>$\beta_{1M} = 400$</td>
<td>$\beta_{2M} = 600$</td>
<td>$\beta_{3M} = 1,300$</td>
</tr>
<tr>
<td>Female</td>
<td>$\beta_{1F} = 150$</td>
<td>$\beta_{2F} = 250$</td>
<td>$\beta_{3F} = 600$</td>
</tr>
</tbody>
</table>

The age by education by gender interaction described in table 14.1 can be understood and dissected like the two by three interactions illustrated in chapter 8. The key difference is that table 14.1 is displaying the slope of the relationship between income and age, and the three-way interaction refers to the way that the slope varies as a function of education and gender.

If there were no three-way interaction of age by gender by education, we would expect (for example) that the gender difference in the age slope would be approximately the

---

1. More precisely, how the slope varies as a function of the interaction of age and gender.
same at each level of education. But, consider the differences in the age slopes between females and males at each level of education. This difference is $-250 (150 - 400)$ for non–high school graduates, whereas this difference is $-350 (250 - 600)$ for high school graduates, and the difference is $-700 (600 - 1300)$ for college graduates. The difference in the age slopes between females and males seems to be much larger for college graduates than for high school graduates and non–high school graduates. This pattern of results appears consistent with a three-way interaction of age by education by gender.

Let’s explore this in more detail with an example using the GSS dataset. To focus on the linear effect of age, we will keep those who are 22 to 55 years old.

```
. use gss_ivrm
. keep if age>=22 & age<=55
(18936 observations deleted)
```

In this example, let’s predict income as a function of gender (female), a three-level version of education (educ3), and age. The `regress` command below predicts realrinc from i.female, i.educ3, and c.age (as well as all interactions of the predictors). The variable i.race is also included as a covariate.

```
. regress realrinc i.female##i.educ3##c.age i.race, vce(robust) vsquish
```

```
Linear regression
Number of obs = 25718
F( 13, 25704) = 411.30
Prob > F = 0.0000
R-squared = 0.1839
Root MSE = 23556

realrinc | Coef.  Std. Err.     t    P>|t|     [95% Conf. Interval]
---------|--------|-------------------|--------|---------|-----------------------------|
  1.female | 1337.13 | 1693.69        0.79 | 0.430  | -1982.61 - 4656.861        |
    educ3  |        |                   |        |         |                            |
     2   | 550.48 | 1782.19         0.31 | 0.757  | -2942.721 4043.673         |
     3   | -11156.1 | 2618.98      -4.26 | 0.000  | -16289.44 -6022.756        |
female#educ3 |         |                   |        |         |                            |
     1 2   | 783.09 | 2021.65         0.39 | 0.698  | -3179.457 4745.655         |
     1 3   | 7657.91 | 3164.29       2.42 | 0.016  | 1455.703 13860.11          |
         | 413.87 | 45.62          9.07 | 0.000  | 324.4515 503.2876          |
age    |        |                   |        |         |                            |
female#c.age |        |                   |        |         |                            |
     1   | -264.98 | 50.66         -5.23 | 0.000  | -364.2746 -165.6937        |
         | 175.85 | 54.75         3.21 | 0.001  | 68.53584 283.1636          |
         | 897.33 | 77.47       11.58 | 0.000  | 745.4851 1049.18           |
edu3#c.age |        |                   |        |         |                            |
         | -80.30 | 60.94        -1.32 | 0.188  | -199.7625 39.15165         |
         | -414.66 | 93.27       -4.45 | 0.000  | -597.465 -231.8473         |
race   |        |                   |        |         |                            |
         | -2935.14 | 273.33      -10.74 | 0.000  | -3470.879 -2399.397        |
         | 185.39 | 95.63          1.94 | 0.050  | -1689.081 2059.872         |
     _cons | 2691.23 | 1495.78       1.80 | 0.072  | -240.5797 5623.039         |
```
Chapter 14 Continuous by categorical by categorical interactions

Let’s test the interaction of gender, education, and age using the *contrast* command below. The three-way interaction is significant.

```
. contrast i.female#i.educ3#c.age
Contrasts of marginal linear predictions
Margins : asbalanced
```

<table>
<thead>
<tr>
<th></th>
<th>df</th>
<th>F</th>
<th>P&gt;F</th>
</tr>
</thead>
<tbody>
<tr>
<td>female#educ3#c.age</td>
<td>2</td>
<td>10.17</td>
<td>0.0000</td>
</tr>
<tr>
<td>Residual</td>
<td>25704</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

To begin the process of interpreting the three-way interaction, let’s create a graph of the adjusted means as a function of age, education, and gender. First, the *margins* command below is used to compute the adjusted means by gender and education for ages 22 and 55 (the output is omitted to save space). Then the *marginsplot* command is used to graph the adjusted means, as shown in figure 14.2.

```
. margins female#educ3, at(age=(22 55))
(output omitted)
. marginsplot, bydimension(female) noci
Variables that uniquely identify margins: age female educ3
```

Figure 14.2. Fitted values of income as a function of age, education, and gender
14.2 Simple effects of gender on the age slope

The graph in figure 14.2 illustrates how the age slope varies as a function of gender and education. Let’s compute the age slope for each of the lines shown in this graph. The margins command is used with the dydx(age) and over() options to compute the age slopes separately for each combination of gender and education.

```
. margins, dydx(age) over(female educ3)
```

Average marginal effects
Model VCE : Robust
Expression : Linear prediction, predict()
dy/dx w.r.t. : age
over : female educ3

<table>
<thead>
<tr>
<th></th>
<th>Delta-method</th>
<th>Number of obs = 25718</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>dy/dx Std. Err.  z  P&gt;</td>
<td>z</td>
</tr>
<tr>
<td>age</td>
<td></td>
<td></td>
</tr>
<tr>
<td>female#educ3</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0 1</td>
<td>413.8695 45.62015 9.07 0.000 324.4557 503.2834</td>
<td></td>
</tr>
<tr>
<td>0 2</td>
<td>589.7192 30.37993 19.41 0.000 530.1757 649.2628</td>
<td></td>
</tr>
<tr>
<td>0 3</td>
<td>1311.202 62.88374 20.85 0.000 1187.952 1434.452</td>
<td></td>
</tr>
<tr>
<td>1 1</td>
<td>148.8854 22.09037 6.74 0.000 105.589 192.1817</td>
<td></td>
</tr>
<tr>
<td>1 2</td>
<td>244.4296 15.25412 16.02 0.000 214.5321 274.3272</td>
<td></td>
</tr>
<tr>
<td>1 3</td>
<td>631.5618 46.90854 13.46 0.000 539.6227 723.5008</td>
<td></td>
</tr>
</tbody>
</table>

Let’s reformat the output of the margins command to emphasize how the age slope varies as a function of the interaction of gender and education (see table 14.2). Each cell of table 14.2 shows the age slope for the particular combination of gender and education. For example, the age slope for males with a college degree is 1,311.20 and is labeled as \( \beta_3\).

Table 14.2. The age slope by level of education and gender

<table>
<thead>
<tr>
<th></th>
<th>Non-HS grad</th>
<th>HS grad</th>
<th>CO grad</th>
</tr>
</thead>
<tbody>
<tr>
<td>Male</td>
<td>( \beta_1 = 413.87 ) ( \beta_2 = 589.72 ) ( \beta_3 = 1,311.20 )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Female</td>
<td>( \beta_1 = 148.89 ) ( \beta_2 = 244.43 ) ( \beta_3 = 631.56 )</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

We can dissect the three-way interaction illustrated in table 14.2 using the techniques from section 8.3 on two by three models. Specifically, we can use simple effects analysis, simple contrasts, and partial interactions.

14.2 Simple effects of gender on the age slope

We can use the contrast command to test the simple effect of gender on the age slope. This is illustrated below.
Chapter 14 Continuous by categorical by categorical interactions

. contrast female#c.age@educ3, nowald pveffects
Contrasts of marginal linear predictions
Margins : asbalanced

| Contrast Std. Err. | t   | P>|t| |
|-------------------|-----|-----|
| female@educ3#c.age |     |     |
| (1 vs base) 1     | -264.9842 50.65695 | -5.23 | 0.000 |
| (1 vs base) 2     | -345.2896 33.98931 | -10.16 | 0.000 |
| (1 vs base) 3     | -679.6404 78.4498 | -8.66  | 0.000 |

Each of these tests represents the comparison of females versus males in terms of the age slope. The first test compares the age slope for females versus males among non–high school graduates. Referring to table 14.2 this test compares $\beta_1^F$ with $\beta_1^M$. The difference in these age slopes is $-264.98$ ($148.89 - 413.87$), and this difference is significant. The age slope for females who did not graduate high school is 264.98 units smaller than the age slope for males who did not graduate high school. The second test is similar to the first, except the comparison is made among high school graduates, comparing $\beta_2^F$ with $\beta_2^M$ from table 14.2. This test is also significant. The third test compares the age slope between females and males among college graduates (that is, comparing $\beta_3^F$ with $\beta_3^M$). This test is also significant. In summary, the comparison of the age slope for females versus males is significant at each level of education.

14.3 Simple effects of education on the age slope

We can also look at the simple effects of education on the age slope at each level of gender. This test is performed using the contrast command below.

. contrast educ3#c.age@female
Contrasts of marginal linear predictions
Margins : asbalanced

<table>
<thead>
<tr>
<th>df</th>
<th>F</th>
<th>P&gt;F</th>
</tr>
</thead>
<tbody>
<tr>
<td>educ3@female#c.age</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0</td>
<td>70.96</td>
<td>0.0000</td>
</tr>
<tr>
<td>1</td>
<td>43.37</td>
<td>0.0000</td>
</tr>
<tr>
<td>Joint</td>
<td>57.21</td>
<td>0.0000</td>
</tr>
</tbody>
</table>

The first test compares the age slope among the three levels of education for males. Referring to table 14.2, this tests the following null hypothesis.

$H_0: \beta_{1M} = \beta_{2M} = \beta_{3M}$

This test is significant. The age slope significantly differs as a function of education among males.
The second test is like the first test, except that the comparisons are made for females. This tests the following null hypothesis.

\[ H_0: \beta_{1F} = \beta_{2F} = \beta_{3F} \]

This test is also significant. Among females, the age slope significantly differs among the three levels of education.

**14.4 Simple contrasts on education for the age slope**

We can further dissect the simple effects tested above by applying contrast coefficients to the education factor. For example, say that we used the ar. contrast operator to form reverse adjacent group comparisons. This would yield comparisons of group 2 versus 1 (high school graduates with non–high school graduates) and group 3 versus 2 (college graduates with high school graduates). Applying this contrast operator yields simple contrasts on education at each level of gender, as shown below.

```
. contrast ar.educ3#c.age#female, nowald pveffects
```

| Contrast                      | Std. Err. | t    | P>|t| |
|-------------------------------|-----------|------|-----|
| educ3@female#c.age            |           |      |     |
| (2 vs 1) 0                    | 175.8497  | 54.7504 | 3.21 | 0.001 |
| (2 vs 1) 1                    | 95.54426  | 26.83611 | 3.56 | 0.000 |
| (3 vs 2) 0                    | 721.4829  | 69.74939 | 10.34 | 0.000 |
| (3 vs 2) 1                    | 387.1322  | 49.38976 | 7.84  | 0.000 |
```

The first test compares the age slope for male high school graduates with the age slope for males who did not graduate high school. In terms of table 14.2, this is the comparison of \( \beta_{2M} \) with \( \beta_{1M} \). The difference in these age slopes is 175.85 and is significant. The second test is the same as the first test, except the comparison is made for females, comparing \( \beta_{2F} \) with \( \beta_{1F} \). The difference is 95.54 and is significant. The third and fourth tests compare college graduates with high school graduates. The third test forms this comparison among males and is significant, and the fourth test forms this comparison among females and is also significant.

**14.5 Partial interaction on education for the age slope**

The three-way interaction can be dissected by forming contrasts on the three-level categorical variable. Say that we use reverse adjacent group comparisons on education, which compares high school graduates with non–high school graduates and college graduates with high school graduates. We can interact that contrast with gender and age, as shown in the margins command below.